UNITED STATES DEPARTMENT OF AGRICULTURE

DEPARTMENT BULLETIN No. 1277

Washington, D. C. ▼ September 18, 1924

INPUT AS RELATED TO OUTPUT IN FARM ORGANIZATION AND COST-OF-PRODUCTION STUDIES

By

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Bureau of Agricultural Economics

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By H. R. Tolley, *Agricultural Economist;* J. D. Black, *Consulting Specialist,* and M. J. B. Ezekiel, *Assistant Agricultural Economist, Bureau of Agricultural Economics*

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INTRODUCTION

The success of the farm business may depend as much upon the details of the operation of the major enterprises as it does upon the combination of the enterprises. Farm-management investigations have placed much emphasis upon the combination of enterprises; whereas the adjustment of the details of each farm enterprise has received relatively little attention, largely because of the difficulties of the necessary economic and statistical analysis. The purpose of this bulletin is to present a method of studying the effect of variations in details of farm practice upon the profitableness of the farm business.

In any one region, farmers differ widely in the methods and practices which they use in handling specific farm enterprises. To

what are these variations due? Partly to differences in conditions peculiar to individual farms, and partly to differences in the adjustments made to existing economic conditions. Examples of the conditions peculiar to individual farms are heavy soils requiring extra labor in plowing and hilly fields preventing the use of improved machinery; examples of differences due to economic adjustments are that some farmers fatten livestock on wheat, whereas others fatten on corn, and that some farmers apply no fertilizer to crops, whereas others apply several hundred pounds per acre.

What is the relation between these variations in methods and practices and the product obtained? Many things are used in growing any farm product—materials, labor, machinery, land—to mention only a few. The net contribution of each of the productive factors must be measured in determining the effect of the variations in any one.

Furthermore, farming is conducted under conditions of diminishing returns. This means that it can not be assumed that 400 pounds of fertilizer per acre will have twice the effect of 200, or that 20 pounds of corn fed to a beef steer per day will produce twice as much gain as 10 pounds. Hence, measuring the average effect per unit of each productive factor is not sufficient; to be adequate, any study must show the effect upon output of each particular variation in input.

Changes in economic conditions frequently cause wide fluctuations in the relative costs of the various productive factors and in the prices of farm products. Hence, the most advantageous adjustment of the practices employed in each enterprise can not be made once for all; they must be constantly altered to meet changing price conditions, if the best results are to be obtained. Furthermore, while we can actually measure only what has already happened, the farmer is concerned not only with what has been most profitable in the past, but with what is the most profitable now and will be in the future. To be adequate, the analysis must therefore be so devised as to assist him in making the shrewdest possible adjustment to coming conditions.

The method of study presented in this bulletin begins with a detailed analysis of the variations in methods and practices (*input variations*), and the effect of these variations upon the product (*output per unit of input*). The basic data thus obtained are then used to determine the *least-cost combination* of inputs by applying value rates to the inputs and outputs at various combinations of the input factors. The least-cost combination is that combination which will produce the product at the least cost per unit of output.

However, since farmers are interested in obtaining the greatest total profit rather than the greatest profit per unit of product, one further step is necessary to take into account the volume of product as well as the profit per unit. Considering the value of the product per unit and the volume of production at various combinations of the input factors, it is possible to estimate the total profit from the enterprise for different input combinations and thus to determine the *most profitable combination* of input factors.

The practices in the handling of a specific enterprise, dairying for example, may be considerably altered without changing the general balance of the farm enterprises or necessitating any change in farm

organization. To the extent that this is true, studies of the individual enterprises may yield valuable information as to profitable readjustments in the handling of specific enterprises. The method presented in this bulletin is believed to be particularly promising for studies within this range.

Beyond a certain point, readjustment of an individual enterprise necessitates readjustment in the whole farm organization. Thus, although it is possible to make material changes in the feeding practices for dairy cattle without affecting the balance of the farm enterprises, it is not logical to consider changing the size of the herd without taking into account the effect of the change upon the other farm enterprises. In this bulletin, the discussion of *combination of enterprises* points out the difficulties in the way of thus determining the best organization, without attempting to solve the problem completely. Until this phase of the analysis is developed further, any conclusions reached by studies of the greatest-profit combination of inputs for given enterprises must be limited to such changes from existing combinations as will not materially affect the balance of the farm enterprises.

The method of study outlined is accompanied by data to illustrate the method and demonstrate the type of statistical analysis required. Since data have never been secured for the purpose of making just the kind of study described, it was necessary to use those collected for other purposes. For this reason many of the statistical examples do not go so far as the theoretical analysis would require. In such cases the line of further attack has been suggested; working out the full analysis with data collected for study by this particular method will show just what modifications in the suggested technique are necessary.

ANALYSES

INPUT VARIATIONS

Farm organization bulletins of the last few years have usually published data as to the physical amounts of the various cost elements "required" in the production of various products. These data are valuable in many ways, but without further analysis they fall short of their highest usefulness. It is believed that the method of analyzing input variations presented here will make them more useful. As will appear later, this analysis is impossible with a small sample or without reasonably accurate data.

The most apparent limitation of "input requirements" as ordinarily presented is that they do not fit all the various conditions under which crops or livestock are produced on different farms. In the same area some farmers are using tractor power for much of their work, and others are relying entirely on horses; some are using 2-row cultivators, and others 1-row cultivators; some are plowing with 4 or 6 horse teams and gang plows, and others are using single walking plows. Likewise, size and shape of fields, kind of soil, topography, and kind of laborer, whether man or boy, affect input. The harvest work varies with the size of the crop. It is impossible, of course, to determine the force of all the variables affecting input, but adjustments should certainly be provided for all the major variables.

A more serious limitation of "input requirements" is that the very concept denies the fundamental principle of diminishing returns, or "diminishing output" as it is now sometimes called, and along with this, the principle of varying costs per unit of product. The principle of diminishing output states that beyond a certain point the output per unit of input varies inversely as the input of the cost element. Or stated conversely, as input increases, the input per unit of output increases. If this principle is true, then there can be no one definite requirement for each unit of product, but instead a different requirement per unit of product as the input changes.[1] As the requirements per unit of output change, so do the costs change, for costs are merely inputs expressed as values.

Variations in Rate of Applying Fertilizer and Manure to Potatoes

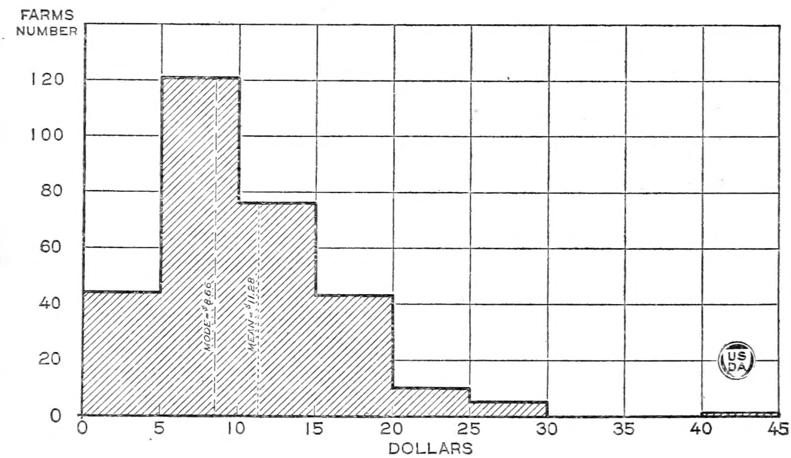

FIG. 1.—Though some farmers applied $20 or $25 worth of fertilizer and manure per acre, more applied from $5 to $10 worth than any other rate. This illustrates the wide differences in practices on farms in the same area. Data from Monroe County, N. Y., for 1913

The actual circumstances are that inputs vary a great deal. A simple case of this is the different quantities of the same kind of fertilizer applied to the same crop by different farmers.

FERTILIZER INPUTS FOR POTATOES

Table 1 and Figure 1 illustrate fertilizer variations for potatoes in New York.

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[1] No constant terminology has been used in presenting "input requirements" in cost-of-production bulletins. The nomenclature varies from "basic cost formula" and "basic acre requirements" to "quantitative requirements for crop production." "Unit requirements" is a clear-cut expression of what all of these phrases mean, namely, certain definite physical quantities of each input factor required to produce a specified unit of output. The whole thesis of this bulletin is that there is actually no such thing as a definite unit requirement for a specified production. It will appear from what follows that there is a range, frequently a wide one, in the amounts of the cost elements which enter into a given product. It is necessary to have some term which expresses the relation of input to output under any specific set of conditions, but which does not imply that this is a fixed relation applying to all conditions. The single term "input" will ordinarily be used, although it does not sufficiently suggest that there is a definite relation between input and output. If occasionally the expression "input requirements" is used, the reader will understand that "requirements" is added merely to suggest this definite relationship.

TABLE 1.—*Variations in the value of fertilizer applied per acre of potatoes on 300 farms in Monroe County, N. Y., 1913* [1]

Value of fertilizer and manure applied per acre	Number of farms
$0. 00 to $5. 49	44
$5. 50 to $10. 49	121
$10. 50 to $15. 49	76
$15. 50 to $20. 49	43
$20. 50 to $25. 49	10
$25. 50 to $30. 49	5
$30. 50 to $35. 49	0
$35. 50 to $40. 49	0
$40. 50 to $45. 49	1

[1] From a survey made by Cornell University in 1913 and 1914. The data pertaining to potato production on these farms were furnished by Dr. E. V. Hardenburg of Cornell University. A complete analysis of the data was published in A Study by the Crop Survey Method, of Factors Influencing the Yield of Potatoes, Cornell University Agricultural Experiment Station Memoir 57, June, 1922. The writers wish to express their thanks to Doctor Hardenburg for his cooperation in furnishing the data for this study.

The most usual or modal rate of application was about $8.66 [2] worth of fertilizer and manure to the acre; but since, as is shown by the figure, there were a few applications at much higher rates, the mean application, $11.28, was much higher.

Why is there this wide variation in the value of fertilizer applied? Some of the reasons may be as follow: (1) Possible variations in the composition of the fertilizers; (2) differences in quality (and value) of the land; (3) differences in present condition of the land; (4) differences in crop rotation; (5) differences in methods of application; and (6) differences in amount of seed to be planted.

Tables 2, 3, and 4 show what relationship there is between value of fertilizer used and the several variables included in the data.

TABLE 2.—*Average input of other factors per acre, by given inputs of fertilizer and manure*

Input of fertilizer per acre	Average inputs of other factors			
	Depth of plowing	Cultivations	Sprayings	Seed planted
	Inches	*Number*	*Number*	*Bushels*
$0.00 to $5.49	7. 33	7. 81	1. 72	10. 59
$5.50 to $10.49	7. 61	7. 64	2. 09	11. 42
$10.50 to $15.49	7. 40	7. 54	1. 71	12. 36
$15.50 to $20.49	7. 61	8. 56	2. 04	12. 07
All farms	7. 56	7. 88	2. 04	12. 14

Table 2 shows no apparent relationship between the input of fertilizer and the input of the other factors, except a slight increase in the quantity of seed as more fertilizer was used. When these rela-

[2] The mode is simply the rate of most frequent occurrence. This lies in the group of $5.50 to $10.49 worth of fertilizer per acre, which covers 121 farms, by far the largest of any group. The midpoint of the group, $8.00, might be taken for the approximate mode; but since there were 76 farms in the next higher group, and only 44 in the next lower, the mode may be more accurately estimated at $\frac{76}{44+76}$ group intervals, or $3.16, above the lower limit of the group; $5.50 + 3.16 gives $8.66 as the mode.

tions are analyzed by multiple correlation,[3] all four factors taken together have a multiple correlation of $R=+0.215\pm0.037$ [4] with the value of fertilizer used, while the quantity of seed used proves to have a net correlation of $r=+0.191\pm0.038$ with the value of fertilizer. On the average, for every increase of 1 bushel in the quantity of seed planted, there was an increase of $0.45 in the value of manure or fertilizer used.

The net correlations with number of sprayings and depth of plowing were both negative, indicating that to the slight extent to which they are correlated, the more fertilizer used, the shallower the plowing and the fewer the sprayings.

From this it is apparent that the application of fertilizer was practically independent of the factors considered, except that it was slightly correlated with the quantity of seed used.

Similar analyses should be carried through to determine the other relations of variations in the value of fertilizer used, and to determine the relations of the variations in the other inputs. Thus, for example, the apparent relation between the quantity of seed and the other factors would be shown by such a table as Table 3. Apparently the input of seed was to some extent related to each of the other cultural factors. Table 4 shows these relations in exact form, measuring the net instead of the apparent correlations.

TABLE 3.—*Average input of other factors per acre, by given inputs of seed*

Input of seed per acre	Average input of other factors			
	Depth of plowing	Cultivations	Sprayings	Value of manure and fertilizer
	Inches	*Number*	*Number*	*Dollars*
6 to 7 bushels	6.85	7.20	1.40	10.10
8 to 9 bushels	7.23	7.19	1.72	8.75
10 to 11 bushels	7.42	7.52	1.55	9.75
12 to 13 bushels	7.62	8.12	2.05	10.67
14 to 15 bushels	7.72	7.92	2.28	11.47
16 to 17 bushels	7.87	8.83	3.25	12.42
18 to 23 bushels	8.57	8.57	3.71	12.71
All farms	7.56	7.88	2.04	11.47

[3] The multiple and net correlation analysis reveals to what extent each factor was correlated with the other, holding all other factors constant and assuming that the relations were all "linear." Thus in Table 2 there seems to be some slight positive correlation between fertilizer and depth of plowing. But the same table shows that the slight increase in the depth of plowing was accompanied by a marked increase in the quantity of seed used. Now the quantity of seed used was positively correlated with depth of plowing as well as with quantity of fertilizer, so that when the effect of the quantity of seed used was eliminated, the "net" relation between fertilizer and depth of plowing became negative. (See the discussion of "Statistical method" for further explanation of the methods of analysis.)

[4] The "±0.037" following the coefficient of correlation is the probable error, and should be read "plus or minus 0.037." This figure means that if the survey had been repeated on other farms in the same area for the same year, and the coefficient of correlation computed for the same variables, the chances are even that it would be $+0.215\pm0.037$; that is, that it would lie between $+0.252$ and $+0.178$. Unless a coefficient of correlation is at least three or four times the size of its probable error not much reliance can be placed on it; in this case it is about six times as large as the error, indicating that there is almost no possibility that the observed correlation is due to mere chance in selecting the farms surveyed. The coefficient of multiple correlation, R, is interpreted the same as the ordinary coefficient of correlation, r; 0 indicating no correlation and 1.00 perfect correlation. However, the r of simple correlation measures only the relation between two factors, while R measures the joint relation between one factor and two or more others; in this case the R measuring the combined correlation of four factors—depth of plowing, number of cultivations, number of sprayings, and seed planted—with value of fertilizer and manure per acre.

TABLE 4.—*Factors affecting rate of planting seed potatoes, determined by net correlation analysis*

On the average, for each additional—	The quantity of seed was increased—	Closeness of the relation
	Bushels	*Net coefficients*
Dollar's worth of manure or fertilizer	0.08	$r=+0.191\pm0.038$
Cultivation	0.13	$r=+0.117\pm0.038$
Spraying	0.44	$r=+0.252\pm0.037$
Inch depth in plowing	0.63	$r=+0.241\pm0.037$

The multiple correlation of all four factors with the quantity of seed was $R=0.428\pm0.032$, showing that the quantity of seed used was related to the cultural factors to a greater extent than was the value of fertilizer applied. Furthermore, the number of sprayings and the depth of plowing had the closest relation to the quantity of seed used, although the other two factors also had some relation.

Carrying the analysis through for all the principal factors of input would give a very complete statement of how variations in the different factors are interrelated. It would show to what extent farmers varied their applications of fertilizer to fit different soil conditions, different applications of seed, etc.; to what extent they changed the quantity of seed used as more fertilizer was applied, different varieties were planted, more cultivations given, different cultural methods used, more sprayings given, or different rotations followed, and so on in turn for each factor. With such information properly presented, a farmer would be able to compare his practices with those of other farmers, not only with the average of all the farmers in the region, but with the average practice of those operating on soils of the same fertility and using the same cultural methods that he used. The accuracy of such comparisons would depend upon the extent to which the factors studied were correlated with the practice under consideration.

In this case the multiple correlation of 0.428 ± 0.032 for the factors related to seed input indicates that there were many other reasons for variations in the seed input than the four factors included. The more of these additional factors that were measured and included in the analysis the closer the multiple correlation would approach 1.00 and the more valuable would be the results.

With such detailed data one could say "the practices on this farm are the same as those on other farms operated under the same conditions," or "differ from the practices on farms operated under the same conditions in (such and such a way)." Collection, analysis, and presentation of data in such a way as to make possible this kind of comparison are the first steps toward making these data fit the actual need of individual farmers.

FEED INPUTS FOR BEEF CATTLE

A somewhat more complicated case is that of feed fed to beef cattle, or dairy cows, or swine. The following analysis shows some of the difficulties for beef cattle. Table 5 shows the variations in feed fed per day for 67 droves of cattle in Nebraska. Figure 2 presents

the same data as a frequency graph. The mean daily rate, 19.15 pounds, is somewhat less than the modal[5] rate, 20.00 pounds.

Variation in Average Daily Rate of Feeding Grain to Beef Cattle

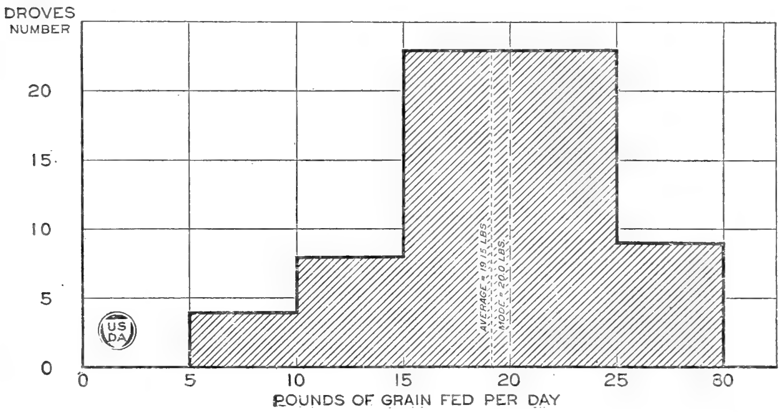

FIG. 2.—Feeding practices in this area (Nebraska) were rather uniform. The great bulk of the farmers were feeding from 15 to 25 pounds of grain per head per day

TABLE 5.—*Showing variations in grain fed to beef cattle, Nebraska, 1920–1921*[1]

Grain fed per head per day	Number of droves
5.0 to 9.9 pounds	4
10.0 to 14.9 pounds	8
15.0 to 19.9 pounds	23
20.0 to 24.9 pounds	23
25.0 to 29.9 pounds	9
Total	67

[1] From data obtained cooperatively by the University of Nebraska, the Bureau of Animal Industry, and the Division of Cost of Production, Bureau of Agricultural Economics, U. S. Department of Agriculture, Preliminary Report, Cost of Fattening Cattle in Nebraska. September, 1921.

According to Table 6, the higher rates of feeding corn tend to be accompanied by a similar heavier feeding of roughage.[6] The following analysis will show whether it can be assumed that the input of roughage follows the same course as the input of corn.

TABLE 6.—*Droves classified according to grain and roughage fed per day*

Roughage per head per day	Rate of feeding grain (per head per day)					Total droves
	5–9 pounds	10–14 pounds	15–19 pounds	20–24 pounds	25 pounds and over	
5 to 9 pounds	3	2	7	8	3	23
10 to 14 pounds	1	5	10	10	1	27
15 to 19 pounds		1	6	4	4	15
20 pounds and over				1	1	2
Total droves	4	8	23	23	9	67

[5] All modes determined as previously explained on page 5.
[6] The hay and roughage fed were reduced to a common measure by counting 2 pounds of corn stover or straw as equal to 1 pound of hay. This is only a very rough measure; for accurate work, each kind of feed should be treated as a separate factor.

It would seem from this table that there is a somewhat definite relation between the quantity of grain fed per day and the quantity of roughage fed. Only one of the droves receiving under 10 pounds of grain per head per day received as much as 10 pounds of hay, and most of those receiving larger quantities of grain also received more roughage. However, the grain fed per day varied both with the weight of the cattle and the length of time they were kept on feed, as shown in Tables 7 and 8. Before the true relation of roughage input to grain input can be determined, the relation with these other factors must be taken into account.

TABLE 7.—*Droves classified by grain fed per day and weight of cattle at start of feeding period*

Average weight at beginning of period	Rate of feeding grain (per head per day)					Total droves
	5–9 pounds	10–14 pounds	15–19 pounds	20–24 pounds	25 pounds and over	
800 to 899 pounds	4	5	7	7		23
900 to 999 pounds		3	6	10	3	22
1,000 to 1,099 pounds			9	4	4	17
1,100 to 1,199 pounds			1	2	2	5
Total droves	4	8	23	23	9	67

TABLE 8.—*Droves classified by grain fed per day and length of feeding period*

Number of days on feed	Rate of feeding grain (per head per day)					Total droves
	5–9 pounds	10–14 pounds	15–19 pounds	20–24 pounds	25 pounds and over	
60 to 89			2	3	3	8
90 to 119			4	7	3	14
120 to 149		2	7	7	3	19
150 to 179		2	6	3		11
180 to 209		2	3	2		7
210 to 239	2	1		1		4
240 to 269		1				1
270 to 299			1			1
300 to 329	1					1
330 to 359	1					1
Total droves	4	8	23	23	9	67

The length of time on feed is highly correlated with the initial weight of the animal; the lighter the animal, the longer it was kept on feed. If these intercorrelations are allowed for by the use of the net correlation analysis, the relations expressed in Table 9 appear. It is now evident that the rate of feeding grain is not correlated with the rate of feeding roughage (fig. 3). When the effect of the weight of the animals and the length of time on feed are eliminated, the correlation between the rates of feeding grain and roughage disappears. The multiple correlation between the three factors and the amount of grain fed per day was $R=0.677\pm0.045$, indicating that the input of corn is largely governed by the two factors, weight of animals and length of feeding period, and, as indicated by the net correlations, is more closely related to the weight of the animal than to the length of time on feed.

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Net Regressions on Three Other Factors of Daily Rate of Feeding Grain to Beef Cattle

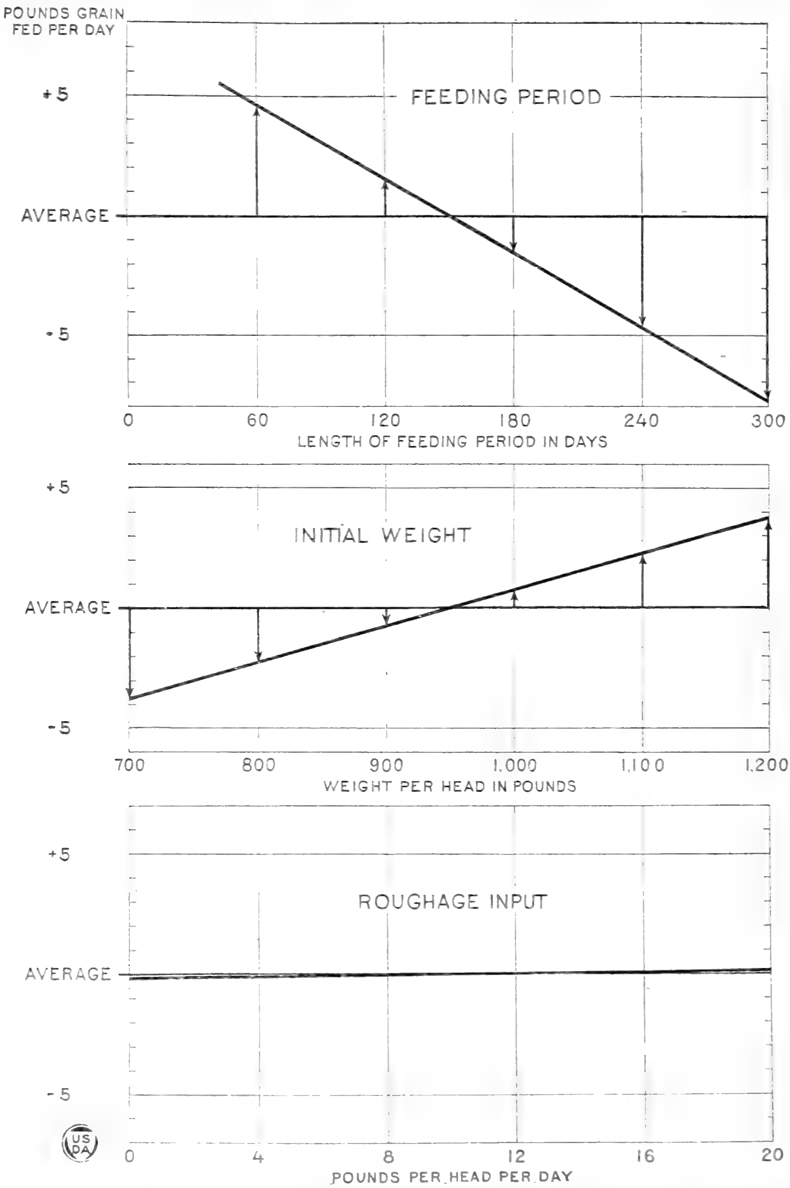

FIG. 3.—The average quantity of grain fed per day was less when the cattle were fed for long periods, and it was heavier for heavy cattle. After the effect of the two other factors had been allowed for, there was no relation between the quantity of grain and the quantity of roughage fed

TABLE 9.—*Net effect of various conditions on grain fed per day*

On the average, for each additional—	Grain fed per day was increased—	Closeness of the relation
	Pounds [1]	
30 days on feed	−1.55	r=−0.259±0.077
100 pounds initial weight	1.51	r=+0.321±0.074
Pound of roughage per day	.005	r=+0.005±0.082

[1] Decrease is indicated by a minus sign (−).

Table 10, calculated from the values in Table 9, shows what would be the average input of grain per day under 20 different conditions. It should be remembered that these 20 inputs are based upon records from only 67 cases; but because of the method of computation, each of these 20 inputs is based upon all 67 cases, instead of upon only the very few droves which happen to fall within any one of the 20 classes.

TABLE 10.—*Computed average input of grain per head per day, by initial weight of animal and length of feeding period*

Days on feed	Weight of animal at beginning of feed (pounds)			
	800	900	1,000	1,100
60	21.5	23.0	24.5	26.0
120	18.4	19.9	21.4	23.0
180	15.3	16.8	18.3	19.8
240	12.2	13.7	15.2	16.7
300	9.1	10.6	12.1	13.6

The values given in Table 10 were calculated from the net regression coefficients of Table 9. These coefficients were combined in a single regression equation as follows:[7]

$$\text{Input of grain per day} = 12.5 - 1.55\left(\frac{\text{days on feed}}{30}\right) + 1.51\left(\frac{\text{initial weight of animals}}{100}\right) + 0.005\ (\text{pound of roughage fed per day}).$$

[7] The first term of the regression equation, 12.5, in this case, is obtained as follows:

$$x_1 = b_{12.34}x_2 + b_{13.24}x_3 + b_{14.23}x_4$$

x_1 being the deviation from the average for the dependent variable, x_2, x_3, and x_4 deviations from the averages for the independent variables, and $b_{12.34}$, $b_{13.24}$, $b_{14.23}$ the coefficients of net regression.

Letting X_1 stand for values of the dependent variable, X_2, X_3, and X_4 for values of the independent variables; and A_1, A_2, A_3, A_4 for the average value of each variable, the foregoing equation can be written:

$$X_1 - A_1 = b_{12.34}(X_2 - A_2) + b_{13.24}(X_3 - A_3) + b_{14.23}(X_4 - A_4)$$

or

$$X_1 - A_1 = b_{12.34}X_2 - b_{12.34}A_2 + b_{13.24}X_3 - b_{13.24}A_3 + b_{14.23}X_4 - b_{14.23}A_4$$

transposing

$$X_1 = [A_1 - b_{12.34}A_2 - b_{13.24}A_3 - b_{14.23}A_4] + b_{12.34}X_2 + b_{13.24}X_3 + b_{14.23}X_4$$

The terms inclosed by the bracket, composed only of the averages and the net regression coefficients, are constant; hence the sum of all those terms is used as a constant "a" in estimating values of x_1, for given values of the other variables, the final equation reading:

$$X_1 = a + b_{12.34}X_2 + b_{13.24}X_3 + b_{14.23}X_4$$

This is the form of the equation in the text.

By substituting any desired values for initial weight of animals, length of feeding period, and roughage fed per day, in this equation, it is possible to compute the most probable rate of feeding grain which would accompany the other conditions, in so far as indicated by the data studied, and subject to the limitations of the assumption of linear relationships.[8]

Figure 3 shows graphically the rate at which the daily input of grain tends to change with changes in each of the three other factors. This is drawn to show the relation of changes in the independent factors to changes from the average of the dependent factor—grain fed per day.

Including additional data in the analysis would show to what extent other factors, such as use of pasture, and quality and breed

Variations in Man Labor Used on Wheat Prior to Harvest

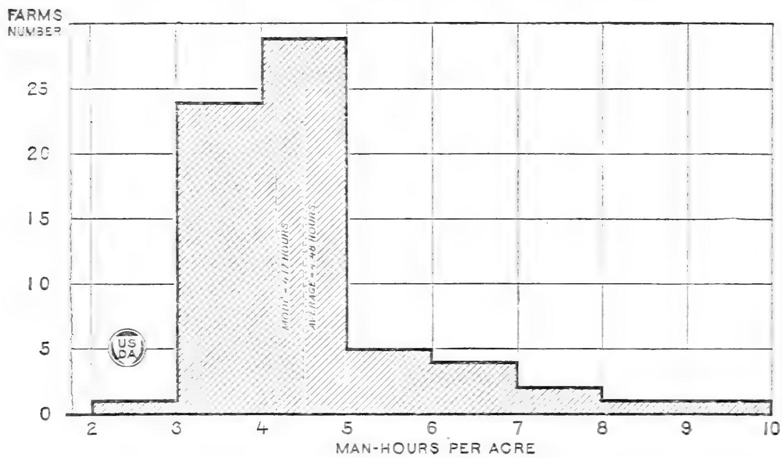

FIG. 4.—Data from survey records in Minnesota. Although some farmers put in as much as 8 or 9 hours per acre, four-fifths of the records fell between 3 and 5 hours

of the cattle, are correlated with the rate of feeding grain. Similar analyses should also be made to show the relations of variations in input of protein concentrates to variations in the input of other feeds.

LABOR INPUTS FOR WHEAT

Variations in labor requirements are more difficult to analyze than variations in any other cost element. There is first the difficulty of measuring the labor itself, for it varies both by type (some being boy labor, some woman labor, some proprietor labor, some hired-man labor, etc.), and by efficiency within each type; and further, its effectiveness is affected by the type of equipment used; that is, 1-row or 2-row cultivators, horses, or tractors. In addition, in many cases there will not be a smooth, continuous series. Increases in man-labor used may represent additional cultivations or different cultural practices. For these reasons, each individual study will present its own problems of analysis. Frequency tables by different cultural methods and by types of labor, subsorting, and possibly partial and even

[8] See p. 43 for discussion of linear and curvilinear relations.

"curvilinear"[9] correlation analysis will have to be employed at times. The following study suggests one method of attack. Table 11 gives the variations in labor per acre on 67 farms in Minnesota in 1919. Figure 4 presents the frequency diagram for the same data. The average input, 4.48 hours per acre, is a little higher than the modal input, 4.17 hours.

TABLE 11.—*Range in man labor used per acre of wheat prior to harvest on 67 farms, Clay and Traverse Counties, Minn., 1921*[1]

Man hours per acre	Number of farms	Man hours per acre	Number of farms
2.0 to 2.9	1	6.0 to 6.9	4
3.0 to 3.9	24	7.0 to 7.9	2
4.0 to 4.9	29	8.0 to 8.9	1
5.0 to 5.9	5	9.0 and over	1

[1] From data obtained by the Division of Cost of Production, Bureau of Agricultural Economics, in a survey study. Reports published in Bulletin 943, U. S. Department of Agriculture: Cost of Producing Wheat, by M. R. Cooper and R. S. Washburn, 1921.

It would be expected that the farmers using more labor per acre would perform more tillage operations. But, contrary to expectations, Table 12 indicates that the farmers employing twice as much labor per acre perform practically the same number of operations as those using less.

TABLE 12.—*Tillage operations performed on wheat land, by hours of man labor per acre*

Man-hours per acre (prior to harvest)	Number of farms	Number of times over: Harrowing, spike-tooth: Before seeding	Number of times over: Harrowing, spike-tooth: After seeding	Number of times over: Disking: Once over	Number of times over: Disking: Lapping one-half
All farms	67	1.66	0.58	0.21	0.16
3.0 to 3.4	11	1.83	.36	.27	.12
3.5 to 3.9	13	1.56	.46	.54	.04
4.0 to 4.4	14	1.58	.79	.21	.05
4.5 to 4.9	15	1.82	.40	.23	.29
5.0 to 5.9	5	1.70	.60	.50	.04
6.0 to 6.9	4	1.50	1.00	.40	.08

Table 13 indicates that there is a tendency for the farmers in the higher groups to use more labor per operation. Thus in the group using 5 to 6 hours per acre an average of about one-quarter more time was spent in performing each operation than in the group using 3 to 3½ hours. Wide variations in the time spent in performing individual operations must be expected. Tables 14 and 15 show the variations for man labor used on plowing and other operations. Similar variations were found for horse labor. Figure 5 presents these data graphically. The variations in the requirements for some operations—cleaning seed, for example—are much more marked than for others, such as plowing. In each case there are variations of at least 100 per cent from the smallest report.

[9] See Statistical Methods for explanation of this term.

Variations in Man Labor Per Acre for Various Operations on Wheat

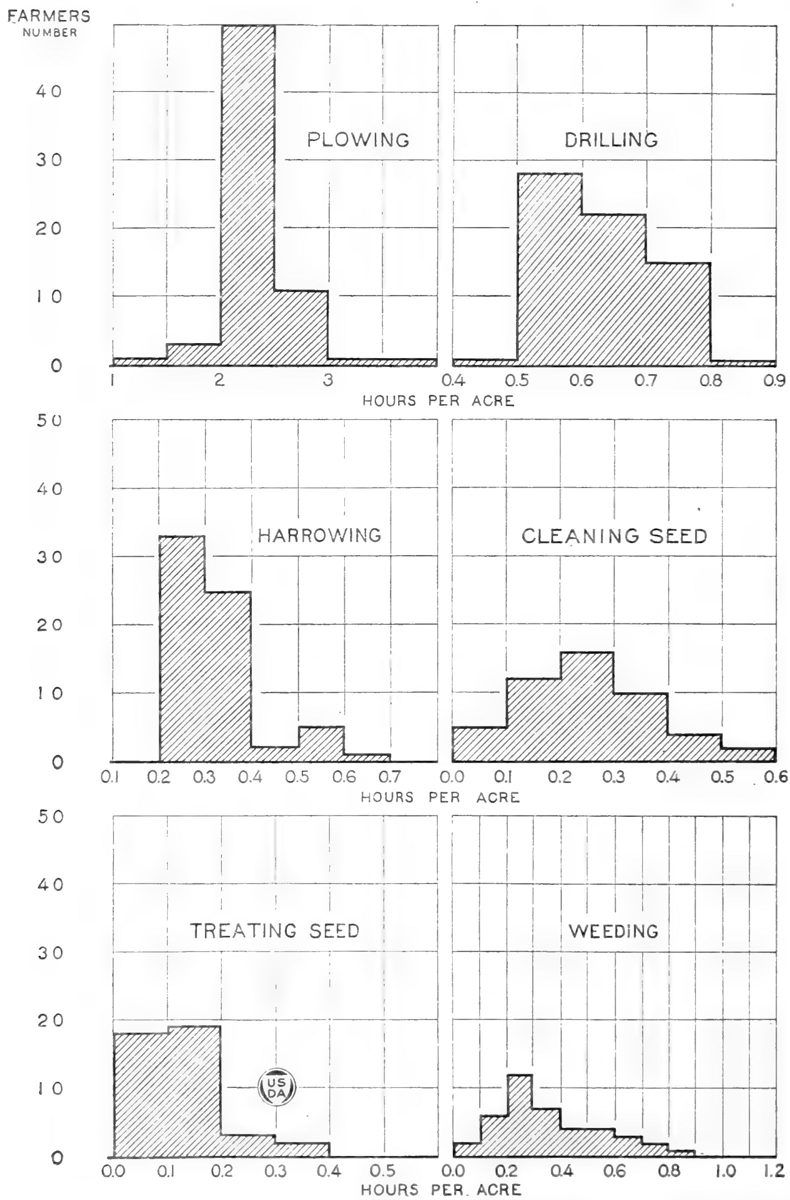

FIG. 5.—This splits up wheat labor to show why some men put in so many more hours per acre than others. Plowing time was very uniform, almost all the records falling between two and two and one-half hours per acre. Drilling labor was somewhat more variable. There were wide variations in the time spent in each of the other operations

TABLE 13.—*Average time used in performing certain operations, by hours of man labor per acre*

Man hours per acre	Farms	Hours per acre, plowing		Hours per acre, harrowing	
		Man	Horse	Man	Horse
	Number				
All farms	67	2.13	11.2	0.28	1.18
3.0 to 3.4	11	2.04	10.8	.25	1.05
3.5 to 3.9	13	2.03	10.8	.28	1.19
4.0 to 4.4	14	2.17	11.5	.28	1.14
4.5 to 4.9	15	2.21	11.2	.31	1.30
5.0 to 5.9	5	2.28	12.5	.28	1.26
6.0 to 6.9	4	2.05	10.8	.23	1.00

TABLE 14.—*Range in hours of man labor per acre used in plowing*

Hours per acre	Farmers reporting each rate
1.00 to 1.49	1
1.50 to 1.99	3
2.00 to 2.49	50
2.50 to 2.99	11
3.00 to 3.49	1
3.50 to 3.99	1

TABLE 15.—*Range in hours of man labor per acre used in performing various operations on wheat*

Hours per acre	Number of farmers averaging each rate				
	Drilling	Harrowing [1]	Cleaning seed	Treating seed	Weeding
0.01 to 0.09			5	18	2
.10 to .19			12	19	6
.20 to .29		33	16	3	12
.30 to .39		25	10	2	7
.40 to .49	1	2	4		4
.50 to .59	28	5	2		4
.60 to .69	22	1			3
.70 to .79	15				2
.80 to .89	1				1
.90 to .99					
1.00 to 2.50					3

[1] Harrowing with spike-tooth before planting.

The next step in the analysis should take up each operation in detail and determine what caused the variations in labor requirements, and should then present the results in such a way that any farmer could tell what would be the most probable labor input for each operation using the particular combination of machinery, horses, and man power that he is using or is planning to use on his farm. Character of soil, size of fields, and topography would also have to be taken into account.

As a further difficulty, probably a large part of the differences in labor input is due to variations in the intelligence and industry of the workers. Just how to measure this remains to be worked out.

It will be seen that, instead of being consolidated into group averages, in all of this analysis each individual report must " stand on its

own feet." Such analysis as here outlined requires both more cases and more precise data than have usually been obtained. Probably the route method, or a combination of route and survey methods, will be necessary to get the full information needed. In the meantime, results from engineering studies of farm machinery can be used to supplement the farm-survey studies.

The use of average hours of labor is but a very vague way of measuring labor input. For both analysis and practical application, the measurement of intensity in the use of labor must be brought down to a solid, definite basis of operations performed, intensity of each operation, and specific causes of variations in labor requirement of each operation.

No doubt a considerable part of the variation is also due to the errors of estimate sure to accompany the survey method; and far from being eliminated by route methods. There is always a considerable amount of "idle time" which will be charged to different tasks on different farms.

OUTPUT PER UNIT OF INPUT

The analysis thus far has shown the variation in inputs, and has indicated the ways in which the inputs of the different factors are correlated. The next step is to discover what output accompanies a given input of each of the elements of input. This is really the fundamental problem in the analysis of input—our knowledge of the relative profitableness of different practices can be no more accurate than our knowledge of the effect of those practices upon the product.

POTATO PRODUCTION ANALYSIS

It might seem at first that all that is necessary for this new purpose is to add two columns to the frequency tables presented in the last section, making them appear as shown in Table 16.

TABLE 16.—*Apparent effect of fertilizer on potato yield*

Amounts of input—value of fertilizer	Number of farms	Output—yield per acre	Output per dollar's worth of fertilizer
		Bushels	*Bushels*
$0.00 to $5.49	44	103.1	26.4
$5.50 to $10.49	121	114.1	15.0
$10.50 to $15.49	76	132.9	10.4
$15.50 to $20.49	43	132.1	7.5

Unfortunately, however, it may be that the farmers using the large amounts of fertilizer are using more or less intensive cultural methods, more labor and less equipment, or vice versa; and the differences in yield may be due to any one or all of these several differences in input. By proper statistical methods (discussed in detail on pp. 40 to 44) it is possible to eliminate the effects of the other inputs, and to determine the net effect of changes in the input of fertilizer alone. Tables 17, 18, and 19 show some of the results obtained from the potato data, as far as it was possible to isolate the factors in the data at hand. These relations are shown graphically in Figure 6.

Net Relations of Three Input Factors to Yield of Potatoes

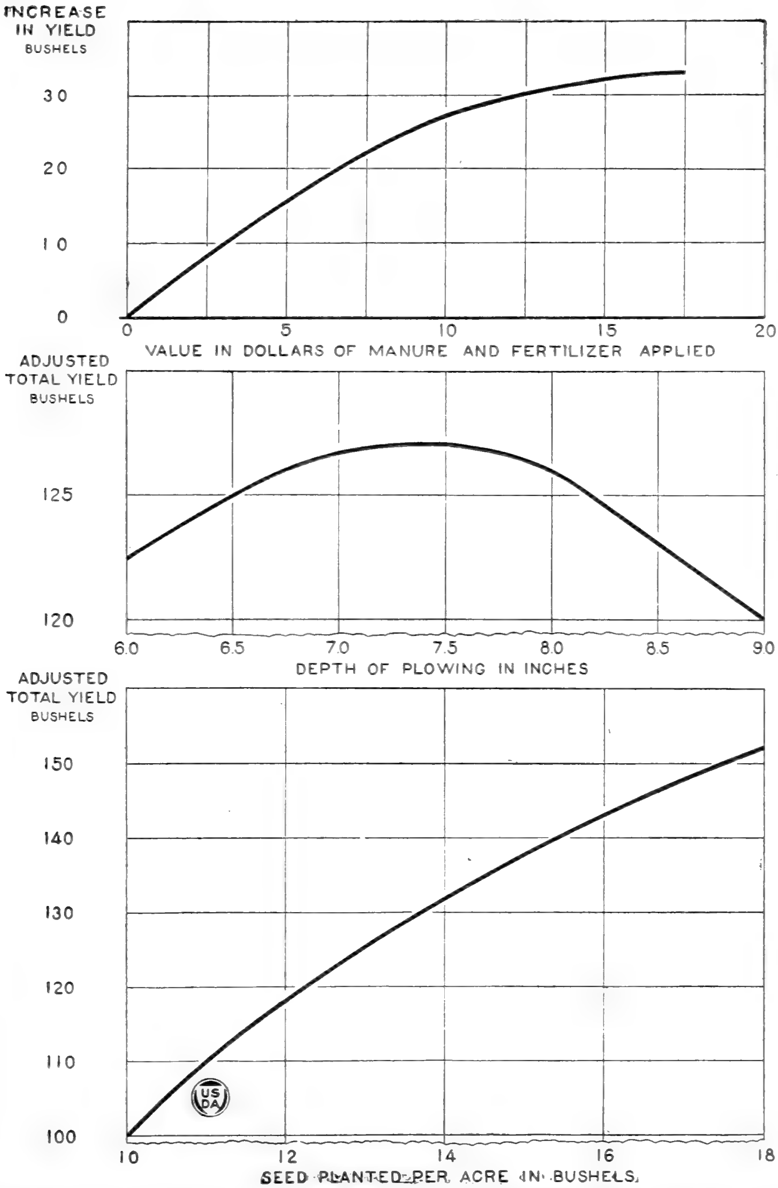

FIG. 6.—Diminishing returns from the use of fertilizer and manure are evident. An optimum depth of plowing is well defined. Planting sufficient seed is essential to good yields. Each of these relations shows up more clearly now that the effects of other causal factors have been eliminated by the partial correlation method

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TABLE 17.—*Net increase in yield of potatoes due to successive increments of manure and fertilizer*

Value of manure and fertilizer applied	Total increase	Increment	Bushels per dollar's input of fertilizer and manure
	Bushels	*Bushels*	
$2.50	8	8	3.2
$5.00	15.5	7.5	3.1
$7.50	22	6.5	2.9
$10.00	27	5	2.7
$12.50	30	3	2.4
$15.00	32	2	2.1
$17.50	33	1	1.9

TABLE 18.—*Relation between depth of plowing and yield of potatoes*

Depth of plowing	Yield after eliminating effects of other factors	Depth of plowing	Yield after eliminating effects of other factors
	Bushels	*Inches*	*Bushels*
6.0 inches	122.5	8.0	126.0
6.5 inches	125.0	8.5	123.0
7.0 inches	126.7	9.0	120.0
7.5 inches	127.0		

TABLE 19.—*Relation between seed planted and yield of potatoes per acre*

Seed planted per acre	Adjusted potato yield	Increase in yield due to increase of 1 bushel in seed input
	Bushels	*Bushels*
10 bushels	100	----------
12 bushels	118	9.0
14 bushels	132	7.5
16 bushels	143	5.5
18 bushels	152	4.5

BEEF PRODUCTION ANALYSIS

The data on beef production were more complete, and an analysis covering more factors was made. Taking into consideration the daily inputs of grain, roughage, and protein concentrates per head, the weight of the animals at the beginning of the feeding period, and the length of the feeding period, it was possible to determine the net effect of each factor upon the gains made. The net gain accompanying an increase of 1 pound in protein concentrates was 0.07 pound of beef. As few droves were fed any protein concentrates, this determination of the relation of protein concentrates to the gain in weight did not have a particularly good basis. For each of the other factors, however, it was possible to determine rather accurately not only the average gain due to it, but the gain due to each individual

quantity in the whole range of variation. Thus, as shown in Table 20, while 10 pounds of grain per day resulted in a gain of 1.15 pounds per head, 20 pounds caused not twice this, but only 1.81 pounds gain, and 30 pounds but 2.10 pounds gain. Similar though not so marked decreasing gains were found for roughage inputs.

Net Relations of Two Input Factors to Gain in Weight in Beef Animals

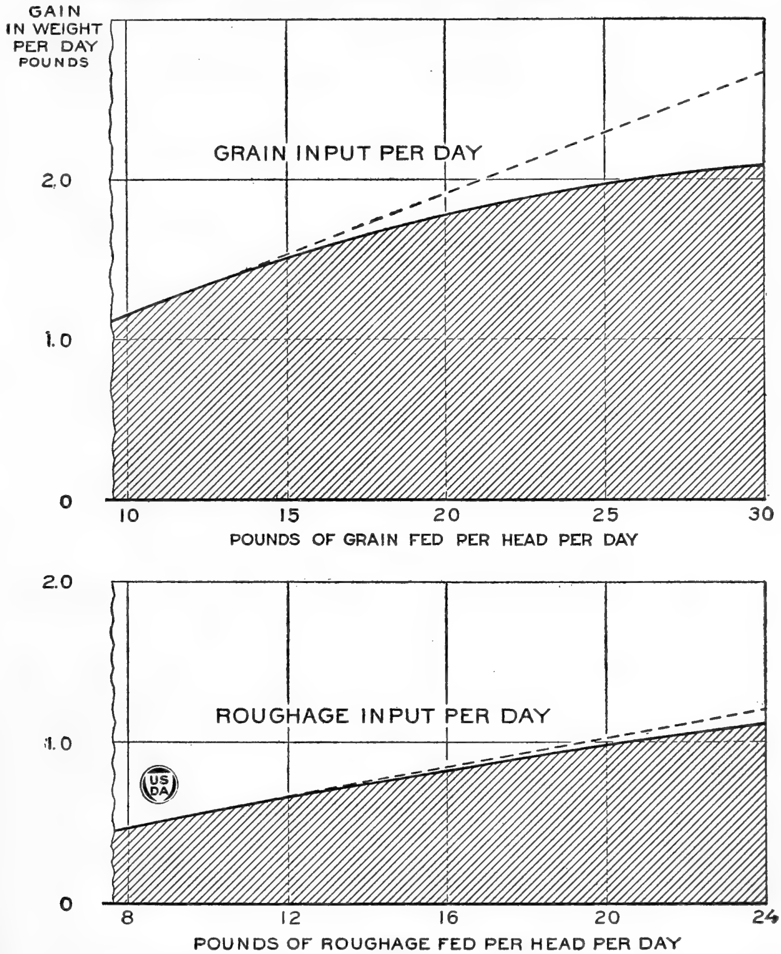

FIG. 7.—The straight lines indicate what would be a constant increase in weight as heavier feeding is practiced. The curves drop off below the straight line because of diminishing returns; the heavier the rate of feeding, the less the gain per pound of feed.

Since in all but one of the 67 droves, pork was produced as a by-product from the same feed, the gain in pork by-product as well as in beef is shown for the grain input, and the effect of the length of the feeding period and the weight of the (beef) animals upon the pork by-product is likewise shown.

The accuracy with which these results measure the factors determining the gains is shown by the fact that when the gain in weight

Net Relation of Feed Input to Feed Requirement for Beef Production

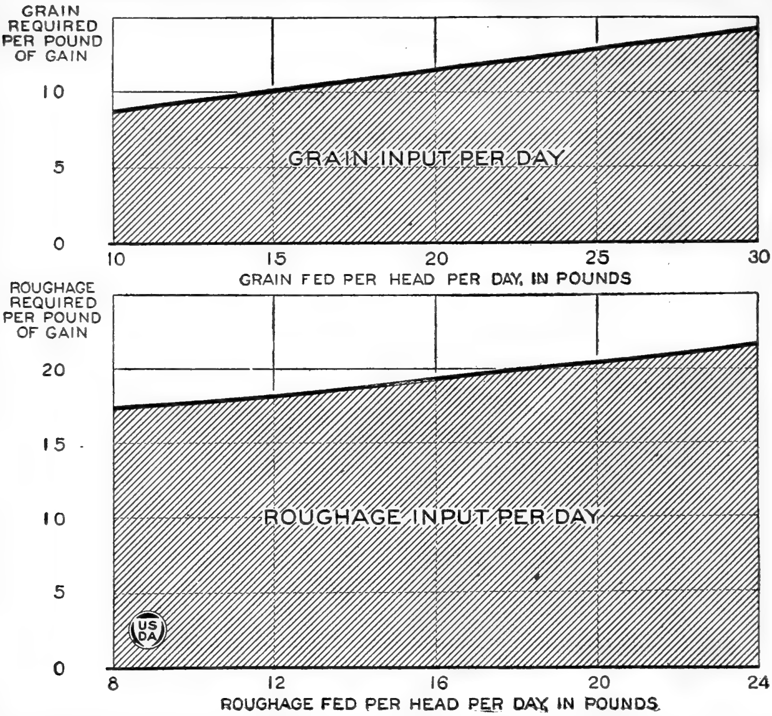

FIG. 8.—Showing how feed requirements per pound of beef change as feeding practices change. The average requirement for steers fed 25 pounds of grain per day is much heavier than for steers fed 10 pounds, other things being the same

Net Relation of Length of Feeding Period and Initial Weight to Feed Requirement for Beef Production

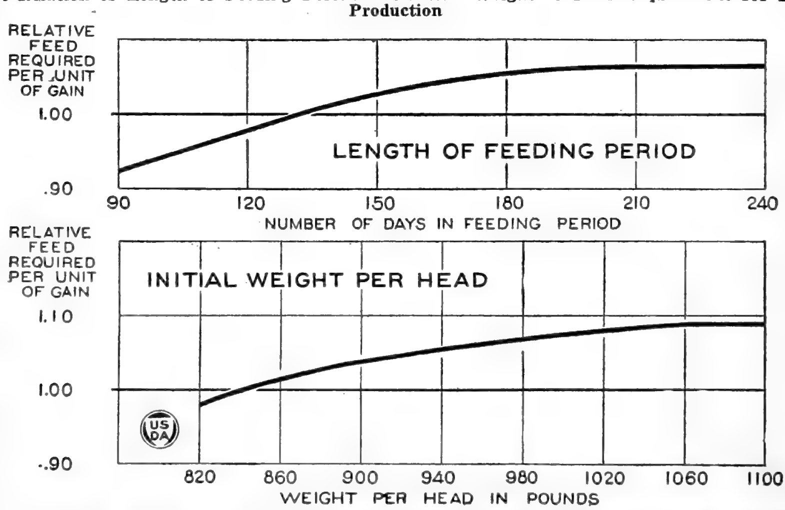

FIG. 9.—Other things besides the quantity of feed per day affect the economy of gains. Gain can be put on much more economically in a "short cram" than over a long period. However, light animals put on gains more economically, so one advantage must be balanced against the other. Sometimes both can be combined, as in the production of "baby beef."

for each of the droves was predicted[10] from the actual inputs, using the relations shown in Table 19, these estimated gains came exceedingly close to the gains actually made.[11]

Figures 7, 8, and 9 show graphically some of the relations given in Table 20. These results show that the fact of diminishing returns must be taken into account in setting up any sort of unit requirements.[12] It takes 8.7 pounds of grain to cause a pound of gain in a steer fed only 10 pounds per day, but it takes 14.3 pounds of grain—64 per cent more—when fed at the rate of 30 pounds per day, whereas the pork by-product, which was 1 pound for each 67 pounds of grain at the lower input, is only 1 pound for each 112 pounds of grain at the higher rate of feeding. Similar wide variations in feed requirements are caused by variations in other feeding practices, as shown.

TABLE 20.—*Net relation of various feeding practices to feed requirements for beef production, and for pork by-product*

A. DIFFERENCES IN THE DAILY INPUT OF GRAIN

Pounds per head per day	Daily gain due to grain		Average grain required per pound of gain of	
	Beef	Pork by-product	Beef	Pork by-product
	Pounds	*Pounds*	*Pounds*	*Pounds*
10	1.15	0.149	8.7	67
15	1.50	.205	10.0	73
20	1.81	.238	11.0	84
25	1.95	.258	12.8	97
30	2.10	.268	14.3	112

B. DIFFERENCES IN THE DAILY INPUT OF ROUGHAGE

Pounds per head per day	Daily gain due to roughage	Average roughage required per pound of gain
	Pounds	*Pounds*
8	0.46	17.4
12	.66	18.2
16	.83	19.3
20	.98	20.4
24	1.11	21.6

[10] Gains were "predicted" by determining from the table how much gain should have been made for the grain and roughage fed and then adjusting these gains according to the length of time on feed and the initial weight.

$$\text{Estimated gain} = \left\{\begin{array}{l}\text{Gain due to corn}\\ +\text{ gain due to roughage}\\ \quad\text{gain due to protein concentrates}\end{array}\right\} \times \left\{\begin{array}{l}\text{Effect of}\\ \text{length of}\\ \text{period}\end{array}\right\} \times \left\{\begin{array}{l}\text{Effect of}\\ \text{initial}\\ \text{weight}\end{array}\right\}$$

Taking the "relative gains per unit of feed" to measure the effect of the last two factors.

[11] The correlation between the actual and the predicted gains was $r = +0.820 \pm 0.027$, and the average error in predicting gains was 12.7 per cent of the actual gains. Separating roughage into its components—leguminous and nonleguminous hay, straw, and fodder—would have made the predictions still more accurate.

[12] The practically linear increase in grain requirement per pound of gain, as more is fed per day, is not due to the fact that this was determined as a linear relation. The output resulting from the various inputs was first determined, and it merely happened that the reciprocals of the curves (fig. 7) gave the nearly linear relation (fig. 8). Straight lines have been drawn into Figure 7 to show the extent to which the curves diverged from linear relations.

TABLE 20.—*Net relation of various feeding practices to feed requirements for beef production, and for pork by-product*—Continued

C. DIFFERENCES IN THE LENGTH OF FEEDING PERIOD

Days on feed	Relative gains per unit of feed [1]		Relative feed required per unit of gain	
	Beef	Pork by-product	Beef	Pork by-product
	Per cent	*Per cent*	*Per cent*	*Per cent*
90	108	138. 5	0. 925	0. 720
120	102	117. 0	. 98	. 850
150	97	101. 7	1. 03	. 785
180	95	92. 0	1. 053	1. 085
210	94	87. 4	1. 065	1. 143
240	94	87. 0	1. 065	1. 150
270	94	87. 0	1. 065	1. 150

[1] Taking the gains predicted by A and B as 100%.

D. WEIGHT OF ANIMALS AT BEGINNING OF FEEDING PERIOD

Initial weight per head	Relative gains per unit of feed		Relative feed required per unit of gain	
	Beef	Pork by-product	Beef	Pork by-product
	Per cent	*Per cent*	*Per cent*	*Per cent*
820 pounds	102	137. 3	0. 98	0. 73
860 pounds	99	118. 5	1. 02	. 84
900 pounds	96	104. 3	1. 04	. 96
940 pounds	95	93. 1	1. 05	1. 07
980 pounds	94	86. 5	1. 06	1. 16
1,020 pounds	92. 5	87. 5	1. 08	1. 14
1,060 pounds	92	89. 2	1. 09	1. 12
1,100 pounds	92	91. 1	1. 09	1. 10

The procedure in applying these figures to a practical problem in feeding is first to compute the gain due to the different feeds used, and then to adjust this output according to the length of period and initial weight of the animals.

Further tables showing the unit requirements under different assumed conditions could be presented. Table 21 is an example.

TABLE 21.—*Estimated grain input per pound of gain for a steer weighing 1,000 pounds at beginning of feeding period, by rate of feeding grain and length of period*

(In addition to gain due to roughage fed)

Daily rate of feeding grain	Length of feeding period			
	90 days	120 days	150 days	180 days
	Pounds	*Pounds*	*Pounds*	*Pounds*
10 pounds	8. 6	9. 1	9. 6	9. 8
15 pounds	9. 9	10. 5	11. 0	11. 8
20 pounds	10. 9	11. 5	12. 2	12. 4
25 pounds	12. 7	13. 4	14. 1	14. 4
30 pounds	14. 2	15. 0	15. 8	16. 1

Similar tables could be presented to show the variations in inputs per unit of output for other cost items—labor, interest, use of buildings—as different feeding practices were followed.

WHEAT PRODUCTION ANALYSIS

Since most of the operations performed on the wheat farms were performed only one, two, or three times, the analysis of the data did not reveal the operation of the law of diminishing returns from successive units in the same way as did the data for operations on potatoes and feed fed to beef cattle. The analysis already made of reasons for variations in the input of labor per acre showed that increased labor was due principally to using more labor on each operation rather than to more operations. Table 22 shows no apparent increase in yield with additional hours per acre.[13] When the effect of number of cultural operations was eliminated by partial correlation methods, the net correlation was practically zero. Accordingly, the tentative conclusion may be reached that the higher labor inputs per operation were due solely to variations in character of the soil, in machinery and equipment used, and in efficiency and composition of the labor, and to errors of estimate by the farmer; and did not represent more thorough tillage; that is, better performance of each operation.[14]

TABLE 22.—*Average yields of wheat by input of man labor per acre*

Hours per acre	Number of farms	Average yields
		Bushels.
3.0 to 3.4	11	8.35
3.5 to 3.9	13	8.85
4.0 to 4.4	14	8.35
4.5 to 4.9	15	8.11
5.0 to 5.9	5	8.02
6.0 to 6.9	4	8.35

The relative effectiveness of labor in the different cultural operations, however, can be determined. Table 23 shows the average input of labor per bushel of wheat for several different operations. Harrowing seems to be a much more efficient way of using labor than disking, as far as the average effect goes. Although there were wide variations in labor input per operation, there was not very much variation in the number of times the different operations were performed. For this reason these data could not show the effect of various individual changes in culture—the number of cases was too small for reliable results. In an area where there were wider variations in cultural practices, it would be possible to determine the gains from individual additional operations of each kind. Thus disking once might be a more efficient use of labor than harrowing the third or fourth time. Where records were available covering a sufficient variety of practices, the same marginal analysis as has been used for potatoes and beef could be applied; that is, curves could be drawn showing the net effect on output of each specific increase in input.

[13] The gross correlation between yield and hours of labor is only $r = +0.038 \pm 0.121$.
[14] Or if higher labor input did represent more thorough tillage, this better tillage had no measurable effect on yields.

TABLE 23.—*Net labor input per bushel of wheat for labor applied in different operations*

Operation	Labor input per additional bushel of yield	
	Man hours	*Horse hours*
Harrowing before seeding	0.67	2.81
Harrowing after seeding	0.64	2.62
Disking	1.12	4.72

One weakness of the foregoing method of analysis is the fact that the various inputs do not vary independently of each other. For example, large doses of fertilizer mean more man labor, horse labor, and expense at the time of harvest. More feed means more man labor. More equipment may mean more or less man labor. Hence, in further development of the statistical technique such relations should be kept in mind. Possibly the results from associated inputs, as well as from individual inputs, might be determined.

APPLICATIONS

The first part of this bulletin has presented the method of analyzing input variations so as to get them in form sufficiently accurate and tangible for use. This part will discuss the applications previously outlined.

THE LEAST-COST COMBINATION OF INPUTS

One important practical need for production data is to enable any farmer to forecast the particular combination of land, labor, capital, feed, livestock, etc., which will make it possible for him to produce at least cost per unit of product. Attempts have been made to determine this on the basis of average inputs, but obviously the results are likely to be misleading. Not until the individual is able to estimate for his farm with a fair degree of accuracy how output will vary year in and year out with variations in input, and how one input will vary with another, can he forecast a reliable least-cost combination. Consequently, methods of the kind developed in the preceding pages are absolutely necessary for any useful least-cost determination.

Given the variations in input per unit of output for all the cost elements as input changes, all that is necessary to determine that combination which produces at least cost per unit is to apply prevailing cost rates to the inputs in the various combinations and locate the least-cost combination.

There will be definite limitations in many cases. For example, one man may be limited by labor, another by capital, another by feed available, and another by acreage. There will probably be a different least-cost combination with each different limitation or combination of limitations.

The method of presenting the data for general use in this way is to construct tables of input variations, such as Tables 17 to 21, and to follow these with tables of costs under various assumptions to serve as guides to farmers in working out their own least-cost combinations. The beef-production analysis will here be used to illustrate the method of presentation.

For the simplest case, it will be assumed that a given feeder can buy or sell all of his corn and hay, so that the quantity fed of either can be adjusted to the exact rate desired. It is also possible, of course, to start out with cattle of different initial weights and fed for periods of various lengths. Only one length of period, 138 days, will be used and one weight of cattle, 847 pounds. A complete presentation would include examples under all of the possible combinations.[15]

Table 24 contains the estimates of daily gains per head under these assumptions.[16]

TABLE 24.—*Probable daily gains per head from feeding various combinations of corn and alfalfa hay to steers of 847 pounds initial weight for 138 days*

(Computed from Table 20)

Corn input per head per day (pounds)	Hay input per head (pounds per day)				Pork by-product (pounds per day)
	8	12	16	20	
10	1.61	1.81	1.98	2.13	0.198
15	1.96	2.16	2.33	2.48	0.273
20	2.27	2.47	2.64	2.79	0.317
25	2.41	2.61	2.78	(1)	0.343

1 Very few droves received as much as 20 pounds of roughage and 25 pounds of grain; hence this combination was not calculated.

Feed is only one part of the costs entering into the product, however. Some of the other costs, such as interest, marketing expense, and general overhead, are independent of the intensity of feeding; others, such as labor, tend to increase with more intensive feeding, though not in the same proportion. In a complete analysis the extent to which these other costs varied with changes in feeding practices should be determined and taken into account in determining the least-cost combination. Since no analysis of the labor requirements in beef production or of the other items mentioned has been attempted with the present example, the computation of the least-cost combination as feed inputs change will have to ignore accompanying variations in labor input. The costs other than feed averaged $11.20 per head for the 67 droves on which this study is based; this will therefore be applied as a flat charge per head in making the next computation.

The following tables show the most probable costs per pound of gain under the assumptions stated with different combinations of feeds, and for different prices of corn and hay.[17] Pork by-product

[15] That is, the illustration applies to only two variables of a five or more variable problem. The complete problem requires securing the best combination as regards input of corn, hay, and protein concentrates, labor, length of feeding period, initial weight of cattle, quality of cattle, etc.

[16] The daily gains were estimated as previously explained. Table 20 shows that 8 pounds of hay per day should result in a gain of 0.46 pound of beef; and 10 pounds of corn, 1.15 pounds. It can further be determined from the figures of Table 20 that animals on feed 138 days would make 100 per cent of these predicted beef gains, and that animals weighing 847 pounds at the beginning would make 100 per cent of the predicted beef gains. Hence the estimated daily gain for beef animals fed 8 pounds of roughage and 10 pounds of corn, under the conditions stated, would be:

$$(0.46 + 1.15)(100\%)(100\%) = 1.61 \text{ pounds},$$

giving the daily gain shown for that particular combination of inputs. The other figures for beef and pork by-products were calculated in a similar manner.

[17] Prices of corn or hay or both may change, because of variations in yield and the like, without any changes in labor, equipment, rent, and similar costs. If all costs rose in proportion, as is approximately the case with price-level changes, there might be no change in the least-cost combination.

is credited at a uniform value of 7½ cents per pound. Varying the assumed value of pork would give slightly varying results.

The figures shown in these tables are the costs that, so far as learned from the analysis, would most probably occur under each specified set of conditions in the area from which the base data were secured. These figures are not presented as the exact statement of what the costs would be on any given farm in the area; but they probably closely parallel what those farm costs would be. That is, for each given set of conditions, the combination which gives the least cost in these estimates is most likely to be the combination which gives the least cost on any particular farm, even though the costs on that farm would not be exactly the same, in dollars and cents, as those shown in the table.

TABLE 25.—*Estimated cost of producing beef per 100 pounds of gain at varying prices of corn, by rates of feeding corn and hay (847-pound steers, on feed 138 days; all costs other than feed $11.20 per head; pork by-products 7½ cents per pound)*

Corn input per head per day	Hay input per head (pounds per day)			
	8	12	16	20
	Cost with corn at 40 cents per bushel, hay at $10 per ton			
10 pounds	$12.95	$12.63	$12.65	$12.70
15 pounds	12.16	11.98	11.96	12.04
20 pounds	11.94	11.78	11.78	11.86
25 pounds	12.65	12.45	12.40	----------
	Costs with corn at 50 cents per bushel, hay at $10 per ton			
10 pounds	$14.07	$13.61	$13.46	$13.45
15 pounds	13.55	13.22	13.11	13.12
20 pounds	13.51	13.23	13.13	13.15
25 pounds	14.51	14.16	14.01	----------
	Costs with corn at 60 cents per bushel, hay at $10 per ton			
10 pounds	$15.17	$14.60	$14.36	$14.39
15 pounds	14.89	14.45	14.25	14.20
20 pounds	15.08	14.66	14.48	14.43
25 pounds	16.16	15.86	15.61	----------

From Table 25 it would appear that, under the conditions assumed, the gain would probably be produced at least cost by feeding 20 pounds of corn per day and between 12 and 16 pounds of hay, with corn at 40 cents per bushel and hay at $10 per ton; by feeding only 15 pounds of corn and 16 to 20 pounds of hay when corn rose to 50 cents per bushel; and by feeding 20 pounds (or possibly more) of hay and something less than 15 pounds of corn with corn at 60 cents.

TABLE 26.—*Estimated cost of beef per 100 pounds of gain at varying prices of hay by rates of feeding corn and hay (847-pound steers, on feed 138 days; all cost other than feed $11.20 per head; pork by-product 7½ cents per pound)*

Corn input per head per day	Hay input per head (pounds per day)			
	8	12	16	20
	Costs with corn at 50 cents per bushel and hay at **$10** per ton			
10 pounds	$14.07	$13.61	$13.46	$13.45
15 pounds	13.55	13.22	**13.11**	13.12
20 pounds	13.51	13.23	13.13	13.15
25 pounds	14.51	14.16	14.01	----------
	Costs with corn at 50 cents per bushel and hay at **$12** per ton			
10 pounds	$14.66	$14.28	$14.27	$14.38
15 pounds	13.95	13.78	13.80	13.93
20 pounds	13.87	**13.72**	**13.74**	13.87
25 pounds	14.83	14.61	14.60	----------
	Costs with corn at 50 cents per bushel and hay at **$14** per ton			
10 pounds	$15.06	$14.94	$15.08	$15.32
15 pounds	14.35	14.33	14.48	14.74
20 pounds	14.26	**14.20**	14.35	14.58
25 pounds	15.17	15.08	15.17	----------

In Table 26, which starts with the same least-cost combination of 15 pounds of corn and 16 to 20 pounds of hay, with corn at 50 cents and hay at $10, the least-cost combination shifts to 20 pounds of corn and 12 to 16 pounds of hay when hay rises to $12 per ton, and to 20 pounds of corn and 12 pounds or less of hay with hay at $14 per ton.

These tables merely bear out the commonplace rule to use less of each feed as it costs relatively more, but they tell not merely which way to change but also how far, taking into account relative gains and (in a complete presentation) such items as weight of animals, length of period, and quality of animals.

Another form of assumption might be made as follows: A man has a given quantity of corn and hay to feed, and a given drove of cattle to which to feed it. What length of period will give the most economical gains? For a specific illustration, the following case is taken from the actual records: On a given farm there were 1,848 bushels of corn and 15⅓ tons of alfalfa hay to be fed to 32 steers, weighing an average of 835 pounds. The total value of the feed was $1,048.88. Table 27 indicates the gains to be expected if the total quantity of feed were fed in periods of different lengths. Now in this case some costs—feed, marketing, etc.—remain the same regardless of the length of the feeding period, whereas others, such as the

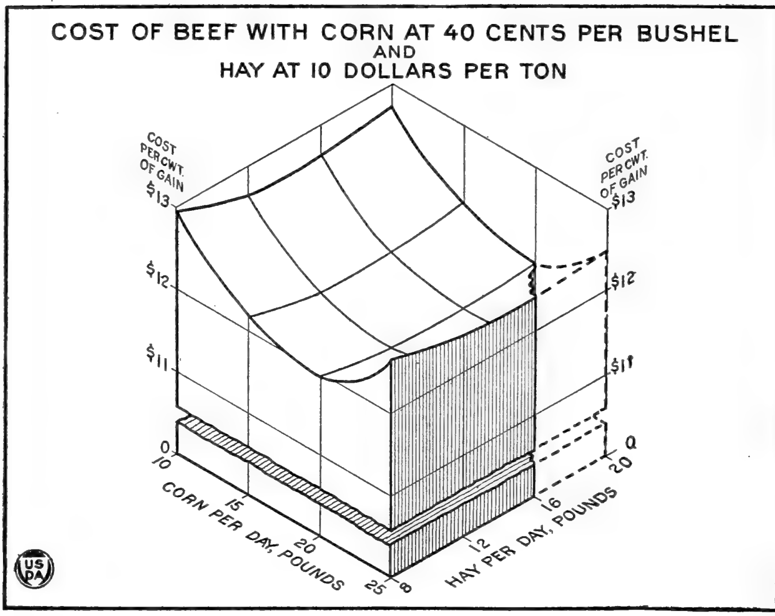

FIG. 10.—With these prices, under the assumptions given in the text, the most economical gains would be made by animals fed about 20 pounds of grain and between 12 and 16 pounds of hay per day

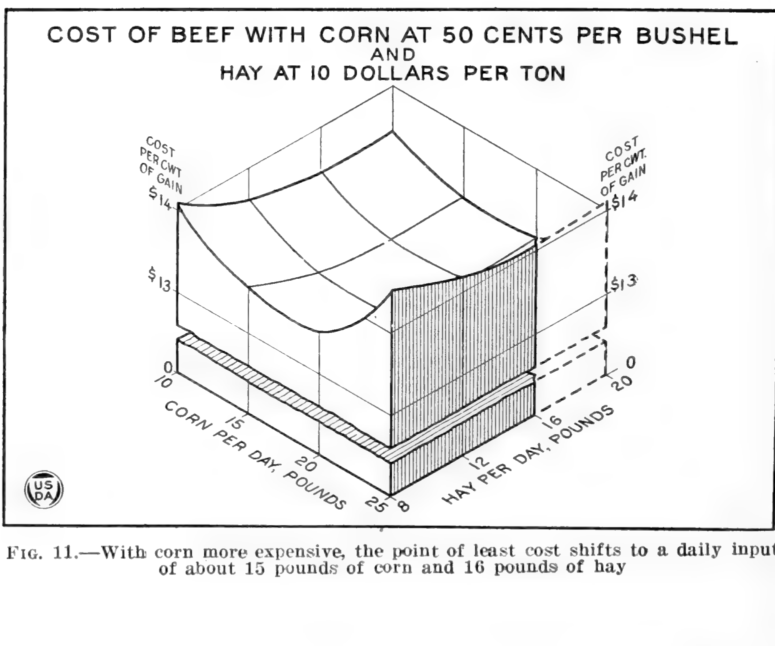

FIG. 11.—With corn more expensive, the point of least cost shifts to a daily input of about 15 pounds of corn and 16 pounds of hay

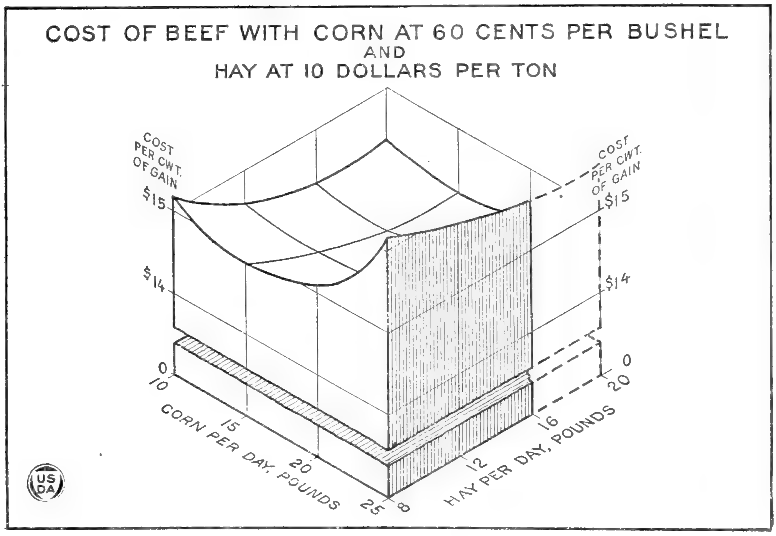

Fig. 12.—Corn is even higher relative to hay than in Figures 10 or 11. The point of least cost has shifted to a daily input of 20 pounds of hay and 15 pounds of corn

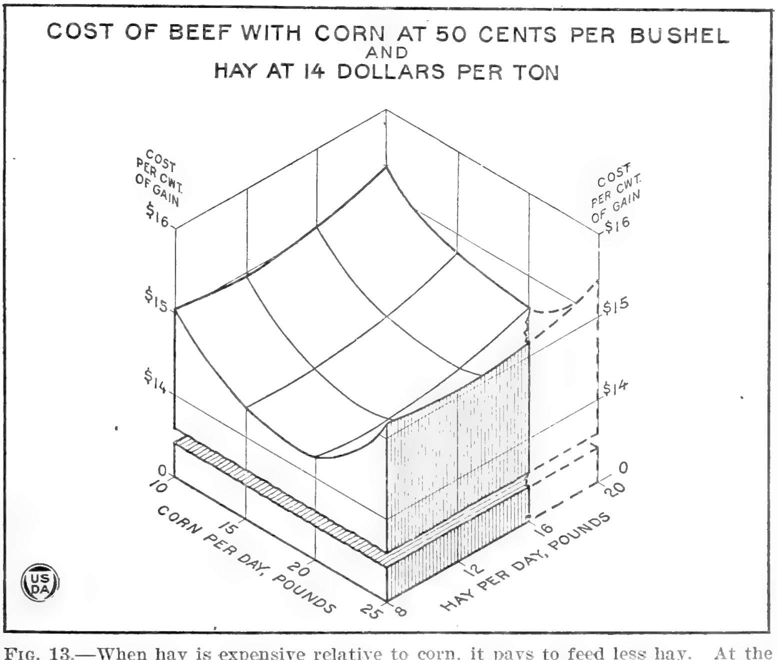

Fig. 13.—When hay is expensive relative to corn, it pays to feed less hay. At the prices given, gain at least cost results from combining a daily input of 20 pounds of grain with but 12 pounds of hay. Compare with Figure 11, where corn is the same price, but hay cheaper

interest charge and labor, vary nearly directly with the length of the period. Still other charges will be intermediate in their relation. In a complete study, the exact relations of all these costs should be determined and used. For this present example, it will be assumed that all costs can be divided into those constant and those varying directly with length of period. For the period for which this drove was actually fed—170 days—the costs assumed to be constant amounted to $35.40 per head, and the others to $6.91 per head. Table 28 gives the costs per pound of gain under these assumptions for the different feeding periods. It appears that the period for which this drove was actually fed—170 days—was not the most economical. This combination represented a rather low input of hay, but since the value of the hay was high compared to the value of the corn, it approached the combination which would have produced gains at the least cost. There were other farmers in this same area who were feeding combinations as intensive as 27 pounds of corn and 16 pounds of hay per day for a 160-day period, or 26 pounds of corn and 9 pounds of hay per day for a 90-day period.

Obviously, if such intensive combinations were analyzed by this same method, many farmers might be found to be producing beef at costs far above the lowest possible, who could profit greatly merely by feeding the same total quantity of feed over a longer period of time, unless marketing conditions dictated the shorter period.

TABLE 27.—*Probable gain per day and total gains from feeding fixed total quantity in periods of different length to 835-pound steers*

(Computed from Table 20)

Length of feeding period	Daily input of—		Daily gains		Total output	
	Corn	Hay	Beef	Pork	Beef	Pork
	Pounds	*Pounds*	*Pounds*	*Pounds*	*Pounds*	*Pounds*
120 days	27.0	8.0	2.45	0.32	294	38
150 days	21.5	6.4	2.04	0.26	306	38
170 days	19.0	5.6	1.92	0.22	326	38
210 days	15.4	4.6	1.74	0.19	365	39
240 days	13.5	4.0	1.56	0.17	374	41

TABLE 28.—*Estimated cost of beef per 100 pounds of gain, feeding a fixed total quantity of feed in periods of different lengths*

(Gains from Table 27)

Length of feeding period	Computation of costs per head				Beef gains	Net cost per 100 pounds of gain
	Fixed costs—feed, etc.	Variable costs—labor, etc.	Deduction for pork credits [1]	Net total costs of gains		
	Dollars	*Dollars*	*Dollars*	*Dollars*	*Pounds*	*Dollars*
120 days	35.40	4.88	3.04	37.24	294	12.67
150 days	35.40	6.10	3.04	38.46	306	12.57
170 days	35.40	6.91	3.04	39.27	326	12.05
210 days	35.40	8.54	3.12	40.82	365	11.18
240 days	35.40	9.76	3.28	41.88	374	11.20

[1] Pork at 8 cents per pound.

These tables serve to suggest the way least-cost combinations can be determined and presented. Each individual product will have its own special problems of calculation and presentation, but the general mode of attack will be the same. This can be stated briefly as follows: Assume various probable or possible combinations of factors, and determine the least-cost combination under each. In presenting the results, state definitely upon what assumptions each computation of costs under various combinations is based and present sufficient data to make it possible to calculate the least-cost combinations under other assumptions.

The foregoing presentation is for a single crop or product. Most farms present a combination of products. Hence several least-cost combinations must be calculated for most farms. The principal difficulty which this presents is that the cost rates for man labor and horse labor vary with the different enterprises, according to whether they conflict or supplement each other, and according to the time of the year. Merely changing the proportions of enterprises may affect the cost rates. Cost rates for land are similarly affected. Least-cost determinations made on the basis of flat rates for labor and land will frequently be seriously misleading.

THE MOST PROFITABLE COMBINATION OF INPUTS

As already suggested, the least-cost combination is not necessarily the combination which will yield the largest profit. Total profit is the product of the profit per unit of output multiplied by the number of units produced; the number of units produced at a higher-cost combination may be enough larger than at the least-cost combination to more than offset the lower profit per unit.

FOR VARIATIONS IN OUTPUT PER PRODUCTIVE UNIT

The variation in output per productive unit may be readily illustrated for the case of a man producing market milk. By buying concentrate feeds he can materially change the production per cow without making any changes in the farm organization. Let us assume that as he increases his production the cost per pound changes, as shown below:

Annual production per cow (pounds)	Cost per 100 pounds
6, 000	$2. 20
7, 000	2. 00
8, 000	1. 95
9, 000	1. 92
10, 000	1. 94
11, 000	1. 98
12, 000	2. 05

Then the least-cost combination will be the one resulting in a production of 9,000 pounds per cow. The production which will give the greatest profit per cow will depend upon the price of milk.

When milk sells at $2.10 per 100 pounds, $2.20, and $2.40, the annual profit per cow will be as shown below:

Production per cow	Total profit with milk selling for—		
	$2.10	$2.20	$2.40
6,000 pounds	Loss.	0	$12.00
7,000 pounds	$7.00	$14.00	28.00
8,000 pounds	12.00	20.00	36.00
9,000 pounds	16.20	25.20	43.20
10,000 pounds	16.00	26.00	46.00
11,000 pounds	13.20	24.20	46.20
12,000 pounds	6.00	18.00	42.00

With milk at $2.10 per 100 pounds, the greatest profit combination would evidently be the same as the least-cost combination—9,000 pounds per year. But if milk were worth $2.20, the greatest profit would be obtained by producing 10,000 pounds per cow; and if it were worth $2.40, by producing 11,000 pounds per cow—even though at 11,000 pounds the cost per 100 pounds would be 6 cents higher than at the least-cost combination.

As long as only variations in the output per cow are thus considered, it is readily apparent that the greatest profit combination can not be a combination producing a smaller volume of output than the least-cost combination, for then both the number of units and the profit per unit would be less than at the least-cost combination. However, as has just been shown, it is possible to obtain the greatest profit at a combination considerably higher in cost per unit than the least-cost combination in those cases where the higher-cost combination is accompanied by a volume enough larger than at the least-cost combination to offset the reduced profit per unit.

The conclusion that as long as the number of productive units—cows, acres, hogs, etc.—is not changed, the greatest profit can not be obtained at a combination less intensive (that is, producing fewer units of output per cow or acre, etc.) than the least-cost combination, is of particular value with regard to the practical use to be made of the analysis. Farmers operating with combinations lower in intensity of production than the least-cost combination are frequently receiving much reduced returns because they produce only a small number of units at a low profit per unit. They can not fail to gain by increasing the intensity of their production to at least the point of least cost if they can do so without decreasing the size of the enterprise as measured by number of cows or other productive units. How far it will pay to increase intensity beyond the point of least cost will depend upon how much increased volume of production offsets decreased profits per unit.

FOR VARIATIONS IN PRODUCTIVE UNITS PER MAN

So far, the discussion has covered the varying of the output per cow or per acre. The greatest-profit combination may also be considered with regard to varying the number of cows or acres per man. This may be illustrated for the case of a man growing a single crop, such as wheat. Let us assume that he is in position to expand his

acreage of this single crop by hiring more labor and renting or buying more land to the point where he will get the largest return for his management. Let us further assume that after a certain point the more land he manages the poorer care he is able to give it, so that the yields are slightly reduced, and the cost per bushel slightly increased, as shown below.

Acres of wheat land managed	Yield per acre (bushels)	Cost per bushel	Acres of wheat land managed	Yield per acre (bushels)	Cost per bushel
25	20	$0.71	125	18	$0.735
50	20	.71	150	17	.755
75	20	.71	175	15.5	.78
100	19	.72	200	12	.82

He would then be producing wheat at least cost per bushel if he grew not over 75 acres. Now if the wheat sold for $0.80 per bushel, he would make $135 by growing 75 acres, $152 by growing 100 acres, and $146 by growing 125 acres. Hence, with wheat at this price, the most profitable combination would be obtained by applying his management to 100 acres, even though the cost per bushel would be 1 cent more at this combination. Similarly, if the wheat sold for $1 per bushel, he would make $435 by growing 75 acres but $625 by growing 150 acres, in spite of the fact that the cost per bushel would be 4½ cents higher at the latter combination.

In this case the farmer would make the greatest profit at a combination considerably less intensive than the least-cost combination. This would be possible only because he would be able to handle a sufficiently increased number of productive units to more than make up the difference in profit per unit. In any given case, how far it will pay to go in expanding the size of the enterprise will depend upon how much the gain from taking on additional acres, cows, or other units is offset by lowered efficiency in the whole production.

In general, so long as there is no change in the size of an enterprise, as measured in physical terms—cows or acres—the most profitable combination can not be less intensive than the least-cost combination. But when there is change in the magnitude of the enterprise, this no longer holds true. Then the total profit from the enterprise must be considered, and the further complication of the effect on the entire combination of enterprises must be taken into account.

FOR BEEF PRODUCTION

The question of the most profitable combination is much more complicated in the case of fattening beef animals than in most agricultural production. The tables previously presented indicate the method of determining the combination which will probably cause the animals to put on flesh at the least cost per pound of gain. But it costs considerably more to put on a pound of gain than to produce a pound of "feeder stuff"; the feeder must make his profit from the "spread" in price which he is able to receive on the weight of the whole animal. Since the final feeding is much more expensive per

pound than the range production, the more the gain that is put on the animal in the feed lot, the higher is the cost to the feeder per pound of the finished animal. The "spread" between the prices paid for the animals as feeders and received for them as beef animals varies with the degree to which they are finished off, for animals with a high degree of finish bring relatively higher prices. The greatest profit per pound of beef sold will be obtained at the point where the difference between the cost of producing the beef (the cost of buying the feeder plus all subsequent costs) lies the furthest below the value of the finished animal. To determine this point exactly, it would not only be necessary to determine the cost (at the least-cost combination) of bringing the animals to each particular degree of finish, but also to determine the price differential for each variation in finish. Knowing both, it would then be possible to locate, for any particular combination of input costs and beef prices, just what combination would probably return the largest profit per pound of beef produced.

Having thus determined the input combination which would result in the largest profit per unit of output, multiplying these profits per pound by the number of pounds which would probably be produced at that combination would give the best approximation for the total return from the enterprise. Comparing the estimated total returns from varying combinations would indicate the most profitable combination for the beef enterprise under the particular conditions assumed.

The problem of the most profitable combination, like the problem of the least-cost combination, is probably not one that can be solved directly by any form of investigation. The probable variations in input per unit of output as input changes can be determined by investigation, but the rest is purely a synthetic process to be performed by each individual producer on the basis of probable cost rates and prices. The variation in input per unit of output as more volume is handled by any manager may be determinable in a general way for any one system of farming; but in the last analysis it is purely an individual problem for each farmer, depending entirely upon his capacity and efficiency as a manager for the particular kind of production in question and the particular kind of cost factors available for his use.

FOR SEVERAL ENTERPRISES SIMULTANEOUSLY

Determining the most profitable combination, like determining least cost, is complicated greatly by the fact that farming usually involves more than one enterprise. The analysis thus far assumes one product only. How shall the highest total return with management applied to several different enterprises, each with its own set of prices, be determined? This bulletin will not attempt to answer this question. There must first be made a careful analysis of somewhat detailed data from several hundred farms in one area, in order to obtain necessary illustrative material of the sort used in discussion of analyses, before attempting to solve the more complicated question of return to management. Such an analysis must take account of volume of output, proportions of enterprises, and input combinations.

Even if the enterprises on a farm were all supplementary—that is, not conflicting as to use of labor and management—so that one could assume that the farmer at any one time was devoting all of his managerial energy to only one of the several products, the problem would still be difficult, for the input combination that meant most profit from one product would not be the same as for the other products. Extra labor could be hired for those products returning greatest profit at the largest input of labor, but of course at higher rates. For horse labor the same amount must ordinarily be used for all. This problem is therefore inextricably involved with the next one to be considered, the choice or combination of enterprises.

It will be remembered, however, that the net gains from the various inputs include whatever effect differences in quantity or quality of management which are correlated with input may have upon output per unit of input. Hence, variations in management are partly included in least-cost inputs as above determined.

THE COMBINATION OF ENTERPRISES

The next problem is to determine what enterprises to combine and in what proportion. If this problem were merely to choose between two or more enterprises, the method would be as follows: On the basis of probable cost rates, and tables or curves of inputs per unit of output properly adjusted to this particular farm, determine the lowest possible cost per unit of product for each product under the conditions existing on this farm, such as available acreage, man labor, horse labor, and management. Compare these costs with the probable prices per unit of the different products. Multiply these different profits per unit of product by the probable number of units of the different products. Choose whatever product promises the greater total profit.

But usually the problem involves, instead, the balancing of several enterprises which are more or less conflicting, supplementary, and complementary. The points of lowest costs and highest profits per unit of product will vary for any product as the proportion of it changes; as the corn acreage increases at the expense of wheat or hay or cotton; or as the number of beef cattle increases at the expense of hogs or sheep; or hay and milk cows at the expense of small grain. The method in such cases is as follows: Estimate the probable lowest cost for all the products under all the likely combinations of proportions of enterprises; compare these costs with the probable prices of the various products; multiply the probable number of units of the various products by the profits per unit. Choose the combination promising the largest total profit from all products in the combination. Include in the receipts from any product the value of all by-products or contributions to other enterprises. Include as costs all contributions from other enterprises.

The choosing between different cropping systems will be handled mostly according to the second of the above methods, since most rotations involve varying degrees of supplementary, complementary, and conflicting relationships.

The difficulties in this problem are, (1) the adjusting of inputs per unit of output as given in tables and curves previously presented to fit variations in the proportions of the enterprises; (2) the adjust-

ing of cost rates, especially for man labor, horse labor, and land, to fit varying proportions of enterprises; and (3) the measuring of the values of contributions by one enterprise to another. These difficulties are so nearly insuperable as to make any highly accurate choosing of enterprises by this method impossible. Since, however, there is such a large element of forecast in the whole determination—forecasts of cost rates, forecasts of prices, etc.—an error in any of the three particulars just named may not be so serious after all.

The fundamental data upon which the study of the combination of enterprises is based is the same input data upon which least-cost and most profitable combinations of input factors are based.

FURTHER APPLICATIONS

There are several further ways in which material studied by the method outlined above may be applied. The study of input variations and of input per unit of output is directly applicable to "operation" problems—to improving day-to-day practices. These results can be presented as production standards which will enable farmers to judge of their own efficiency and indicate what changes will make their production more efficient and profitable; and they can also be used in the choice of farm practices, to determine the advisability of using specific practices, such as fall rather than spring plowing and pasture in preference to dry-lot feeding.

The best organization for the whole farm business may be studied from several different angles. Detailed plans can be made for a given farm for a given year and the most probable net return under each plan estimated. In so planning a farm business it is essential to be able to estimate probable costs and returns. The use of properly analyzed input and output data will lessen the error of estimate in such planning. Or the problem may be approached from the viewpoint of determining what modifications should be made in the existing combinations of enterprises. For this purpose the least-cost or greatest-profit combinations of inputs for the different enterprises may be used as the basis of cost indices which, together with forecasted prices, will give advance information as to probable trends in the relative profitableness of competing enterprises when they are conducted in the proper economic combinations.

Data analyzed by this method offer new material for the study of the relations between production costs and prices. The least-cost and greatest-profit combinations at various prices of the input factors may be used to throw light on the elasticity of supply of different products, to aid in determining to what extent farmers increase or decrease their production with changes in the prices which they receive. It is only to the extent that farmers do make such changes that the costs they incur in their production affect the price they receive for their products. Volume produced, not cost of production, is the active force on the supply end of the supply-and-demand equation.

PRODUCTION STANDARDS

No use for production data is more important than that of supplying individual producers with standards by which they may judge the efficiency of their own performance. Tables 1 to 22, with some further analysis, supply this need.

One type of standard, however, needs to be related only to output and input. This standard may properly be called the standard of efficiency, or output per unit of input. It is thus a purely physical standard. With tables and curves, such as presented under "Analyses" as guides, a producer should be able to determine the output per unit of input obtained upon the average from the same combination of inputs as his own under conditions such as prevail on his farm. If he does not attain this, it must be because of less efficient labor, equipment, livestock, crop varieties, and the like. The average efficiency may thus serve as a standard.

The types of standards usually presented, however, are the "unit requirement" standards and the "standard costs." A general average input is the usual form for a unit requirement standard. Obviously, it is not well fitted to the purpose. First of all, it must be broken up into several standards, according to differences in cultural or feeding practices and differences in conditions affecting production. It is impossible and useless to provide standards for all possible variations in practices or conditions; but all the more important and typical variations should be provded for, so that a farmer can make a reasonably close fit to the conditions on his particular farm.

Moreover, a standard can not be stated without reference to those inputs which give the largest return per unit of input, or to the least-cost and most profitable combinations of input, or to the most profitable combinations of enterprises. An average of a large number of inputs, many of which are far from being most advantageous, surely can not without further analysis be set up as a standard which others should seek to attain.

Two procedures are possible for overcoming this difficulty. One is to forecast average cost rates and average prices for the next 5 or 10 years, to calculate on the basis of these the most profitable combination of inputs and enterprises, and to set up the inputs of the various cost elements accompanying such combinations as the standard inputs, with the idea that producers who most nearly attain to these during the next 5 or 10 years will probably make the largest profits. This also amounts practically to setting up standard combinations of enterprises.

The other procedure is to forecast cost rates and prices for next year only and set up standard inputs for next year only.

Following either procedure, it will be necessary to set up the standards in such a way that they can be used to fit individual farm variations.

Part of the trouble with production standards in the past has been the attempt to express them in the form of "standard costs." Such standard costs have very little use. In the first place, they are out of date before they are published. In the second place, they combine in one figure, input, cost rates per unit of input, and output, all of which must be known as separate facts before the standard cost can be applied to a particular farm. Sometimes they merely combine input and cost rates per unit of input, as in standard costs per acre, etc. This only makes them somewhat more usable. A better plan is to present only inputs and outputs per unit of input, and then let the producer apply his own cost rates if he sees any purpose in so doing. Each producer will need to adjust the input and output data as closely as possible to his particular farm.

When "standard costs" are obtained by merely applying uniform cost rates to all farms alike, as is frequently done for horse labor and man labor, they do not confuse input, output, and cost rates as when all three vary; but they do conceal the fact that the variations are due solely to input and output variations.

There are other production standards of a simpler kind which are valuable, such as bushels of corn husked per man per day, acres of wheat harvested per outfit, average hours of horse labor per day. For a few purposes "standard cost rates" are also worth while. Discussion of these standards is omitted for lack of space.

CHOICE OF FARM PRACTICES

A farmer ordinarily has a considerable range of choice as to cultural methods, equipment, feeding rations, and the like. Given proper data as to outputs per unit of input for each of two alternative practices, all that the farmer needs to do is to apply probable cost rates to the input data for each and see which turns out a unit of output at lowest cost at its least-cost combination. If, for example, he knows how much input of each of two possible balanced rations goes with various outputs of milk (or butterfat or both combined) at least-cost combination on his farm, it being assumed that the inputs of the other cost factors are the same, he merely has to apply the cost rates for the two rations to the units of input and locate the least-cost combination for each under the limitations prevailing upon his farm.

The principal difficulty with this procedure is that the inputs of the other cost factors do not always remain the same—one ration may take more labor than the other, one cultural practice may take more man labor, horse labor, and equipment than the other. In the case of comparing farms with and without tractors, the whole farm organization must be taken into account, for the horse-labor cost rates charged to the enterprises using horse labor in either case are seriously affected thereby, and the acreage of oats and hay as feed crops for horses may be reduced. Most of these difficulties, however, can be handled in a fairly satisfactory way.

PLANNING A FARM BUSINESS

Considerable use has always been made of "requirements" as to quantities of feed necessary per head of livestock, labor hours per acre of various crops, and other similar data, in attempts to plan a well-integrated farm business. It will now be obvious that there can be no definite "requirement" for most of these things. What is needed instead are data showing how outputs change per unit of input, and vice versa, so that the input and output data used will more accurately forecast what will be the situation under the proposed plan of organization.

Much of the planning of the farm business must be for a considerable period of time; for example, the acreage, the buildings, the farm layout. All that can be done for such features of the plan is to guess at probable cost rates, yields, and prices and figure on the basis of these.

Other parts of the plan, such as the number of cows to be kept, the acres of corn to grow, can be calculated at much shorter range. Plans based on input variations and fitted to the particular conditions of the farm should give a much closer forecast of actual results than plans based on inflexible requirements.

COST INDICES

For certain purposes it is desirable to have an index figure showing how the costs of various farm products have changed in the past, what they are now relatively, and, in some cases, what they are likely to be in the near future. The plan usually suggested for making such indices is to apply cost rates at the time to unit requirements of the various cost factors, and thus to assemble a series of total cost figures which can, if desired, be reduced to relatives on the basis of some one year's costs.

It must be realized that the purpose of such indices is not to show what costs should have been, but rather what they were and are. At any one time most farmers are producing at other than the least-cost or most profitable combination of inputs; also the limitations prevailing on each farm make the least-cost and most profitable combination different for different farms. For an index of past changes, therefore, some summary expression of actual inputs is what is wanted.

If the index, however, is to be in any way balanced against price indices for the same products, it should not be based upon the modal or mean inputs, but upon the inputs of the marginal group of producers, say, all those whose product falls between the upper sixty-fifth and ninety-fifth percentiles. This will give a group large enough for the necessary stability of the index, and yet reasonably responsive to changes in conditions of production among the marginal producers. The producers between these two percentiles will, in some cases, be the poor farmers producing all products at high costs; but they will mostly be better farmers growing so large an acreage of this particular crop that their alternative costs are high. The upper 5 per cent, it might be 10 per cent, are excluded because they probably represent abnormal conditions, such as accidentally low yields and the like.

If indices are to be used in forecasting necessary price, the index of marginal inputs is the better one to use, although it would be desirable for such a purpose if the inputs of all-round poor farmers with no better alternatives could be kept out of the group average.

For some purposes, cost indices based on least-cost inputs would be more useful.

The principal difficulty in constructing cost indices is that of cost rates. If flat rates are applied to all labor, for example, the labor element will be overweighted or underweighted in the index. Charging family labor at hired labor rates has the same effect. Similarly the land may be overweighted or underweighted by flat rent charges for all crops.

Will such indices be of use to farmers in choosing combinations of enterprises? If a series of cost indices were projected into the coming year on the basis of probable cost rates, farmers could note any trends, compare them with indices of trends in prices of the

various products, and undoubtedly as a result make wiser decisions than they are now making and probably could make on the basis of cost data now available. In fact, it is likely that this is the form in which aid will have to be brought to the large majority of farmers for a long time to come. Such indices should be constructed as soon as possible for all the important systems of farming in the United States.

ELASTICITY OF SUPPLY

To what extent and how can input data be used in forecasting the supply which will be forthcoming at any given price? As already suggested, cost indices may indicate necessary relative changes in price. But for tariff making and other purposes it is sometimes desired to have absolute data. It has been assumed at times that one can determine from an array of costs what amount will be produced at any particular price. Unfortunately, the problem is not so simple as this. The new supplies forthcoming after a rise in price come more from farmers already producing at a profit than from the marginal producers. If input data are to be used in this way, there must be an analysis of cost rates and profits to go along with them that will show how cost rates and profits vary with the proportions of enterprises in the enterprise combination.

STATISTICAL METHODS

PROPER THEORETICAL ANALYSIS ESSENTIAL

The statistical analysis of any given problem can be no better than the theoretical considerations upon which it is based. An economic basis for the analysis of farm production has been presented; but before proceeding to the statistical analysis, it is necessary so to outline the procedure as to take account of the biological relations involved. The production of agricultural commodities involves the life-process of plants and animals; variations in input affect output only through physiological reactions. For this reason the theoretical framework of the analysis should be based on all available technical and experimental knowledge of the productive process, as well as upon the economic considerations. Statistical method is but a tool; it enables one to test or measure any specific relation or set of relations, but it can not indicate the direction or meaning of the relations.

THE NATURE OF THE PROBLEM

The methods employed in the analysis of farm data to determine the relations between the various input factors and the resulting output must be such as to show the net contribution to the product of each different input. Since the average unit contribution of a given input tends to be different for different total quantities of input, that is, the increases in output tend to follow the law of diminishing returns, the method must further show the contribution of additional units of each input as well as the total or average contribution.

The first of these conditions requires the use of multiple correlation, whereas the second requires some method of measuring curvilinear relations both between pairs of variables and among many variables.

MULTIPLE CORRELATION

The correlation of two variables (gross or simple correlation) determines what value of one variable may be expected to accompany a given value of the other, and further gives a measure of how close the relation is.

The case of feeding pigs will be used for an illustration. Figure 14 is a dot chart showing the average quantity of corn fed per pig on each of 31 farms,[18] and the gain in weight made on the average for each farm. It is at once evident that the more corn the hogs received, the more gain they made. Determining the regression of weight on corn fed, we find the straight line indicated, which shows that on the average each additional bushel of corn fed the hogs was accompanied by a gain of 9.4 pounds in weight.[19] However, the pigs received other feed in addition to the corn—oats, mill feed, and tankage being most common. The gains made were due, therefore, only in part to the corn, the remainder being due to the other feed. Further, the average gain which accompanied the consumption of a certain amount of corn was not the net gain due to corn; it was the gain due to the corn plus the accompanying feeds.

The problem is to find, not how much gain accompanied the consumption of a given amount of corn, but how much of that gain was due solely to the corn fed, eliminating gains due to the other feeds. Multiple correlation is the only method which can give us this result. Considering the corn, oats, tankage, and mill feed used, an analysis by multiple correlation gives the following information:

(1) The average gain in pork per additional bushel of corn fed, eliminating the gain due to the oats, tankage, and mill feed which was fed along with the corn—(net regression of pork on corn).

(2) The average gain in pork per additional bushel of oats fed, eliminating the gain due to corn, tankage, and mill feed—(net regression of pork on oats).

(3) The average gain in pork per additional hundredweight of tankage fed, eliminating the gain due to corn, oats, and mill feed—(net regression of pork on tankage).

(4) The average gain in pork per additional hundredweight of mill feed fed, eliminating the gain due to corn, oats, and tankage—(net regression of pork on mill feed).

(5) A formula for estimating the pork production to be expected from feeding any given combination of corn, oats, tankage, and mill feeds (the net regression equation), and a measure of the probable accuracy of such estimates (the multiple correlation of pork with all four feeds).

The use of this method secures results from the farm data comparable with those obtained under experimental conditions. In an experiment all factors are held constant except the one whose effect it is desired to measure. This can not be done under farm conditions.

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[18] From records secured in Illinois and Iowa by the Division of Cost of Production, Bureau of Agricultural Economics, U. S. Department of Agriculture.

[19] References on simple linear correlation:

Tolley, H. R., and S. W. Mendum. Method of Testing Farm Management and Cost of Production Data for Validity of Conclusions, U. S. Department of Agriculture Circular 307, 1924.

King, Willford I. The Elements of Statistical Method, pp. 197–202 · 13–215. 1919.

Kelley, Truman L. Statistical Method, pp. 179–185. 1923.

Yule, G. Udny. An Introduction to the Theory of Statistics, pp. 157–209, 352; 1922. (Sixth edition.)

The use of multiple correlation does eliminate the variations in other factors and obtain results from which the effects of all other (measurable) factors have been eliminated. Such results reveal the

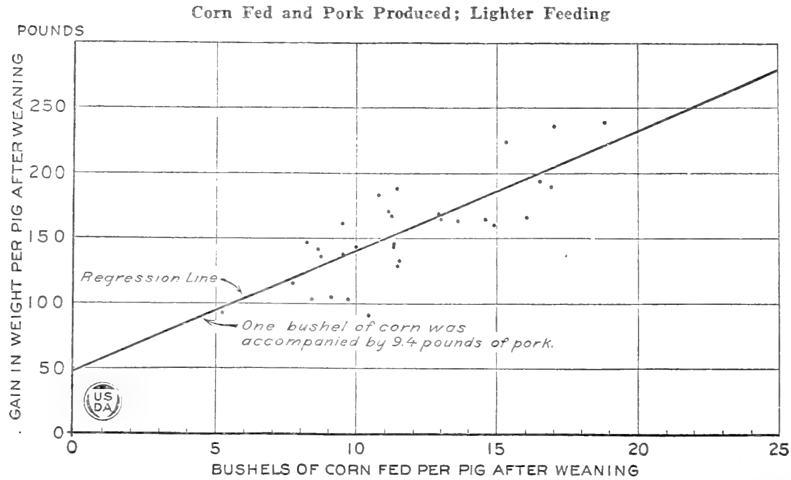

FIG. 14.—These pigs received less than 12 bushels of corn on an average. The regression line, obtained by gross correlation, shows an increase of 9.4 pounds of pork for each bushel of corn

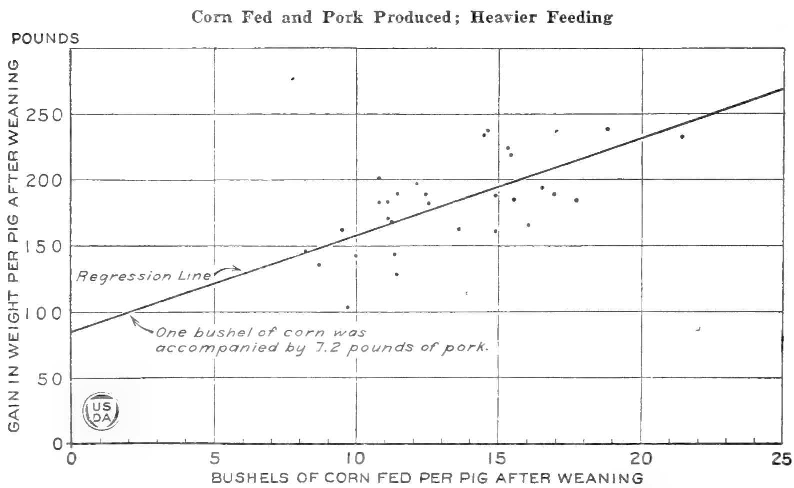

FIG. 15.—These pigs were fed about $13\frac{1}{2}$ bushels of corn apiece. They show only about three-fourths as much gain per bushel as do the pigs on the lighter feeding

net effect in actual farm practice of variations in an individual input practically as accurately as those obtained in the artificial setting of controlled conditions in laboratory or test-plat.[20]

[20] References on multiple correlation:
Tolley, H. R., and M. J. B. Ezekiel. A Method of Handling Multiple Correlation Problems, Journal of the American Statistical Assoc., Vol. XVIII, New Series, No. 144, Dec., 1923.
Kelley, Truman L. Statistical Method, pp. 279 to 295. 1923.
Yule, G. U., id., p. 229–253.

CURVILINEAR CORRELATION

The fact that output does not increase in constant proportion to increases in input, the economic law of diminishing returns, is especially true of agricultural production. Obviously, then, if all but one input factor were held constant, and regularly increasing quantities of that one factor were applied, the increases in yield would not be exactly proportional to the increases in that input but would be progressively smaller and smaller. The relation between the input and the output is hence not linear; each unit increase in input does not produce the same increase in output. Instead the relation is curvilinear; the increase in output is different for each specific increase in input.

How can this relation be measured statistically? Simple or multiple linear correlation will reveal the average effect of variations in each input factor. Figures 14, 15, and 16 illustrate the limitations

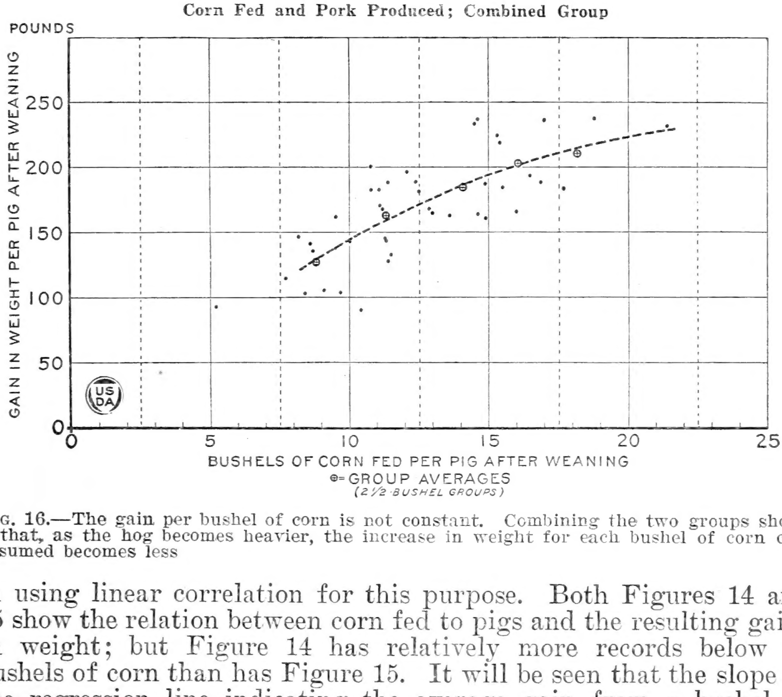

FIG. 16.—The gain per bushel of corn is not constant. Combining the two groups shows that, as the hog becomes heavier, the increase in weight for each bushel of corn consumed becomes less

in using linear correlation for this purpose. Both Figures 14 and 15 show the relation between corn fed to pigs and the resulting gains in weight; but Figure 14 has relatively more records below 12 bushels of corn than has Figure 15. It will be seen that the slope of the regression line indicating the average gain from a bushel of corn is steeper in Figure 14 than in Figure 15, each additional bushel of corn being accompanied by an average gain of 9.4 pounds in the former group, but only 7.2 pounds in the latter group.

Figure 16 shows the groups combined.[21] The farms were sorted according to the quantity of corn fed, and the average corn and pork for each of these groups indicated on the chart. A curve has been drawn, free-hand, approximately through these averages to indicate

[21] In order to insure comparability, the farms shown in Figures 14 and 15 were both selected from those shown in Figure 16, the lower group by discarding half of the farms with corn inputs above the average and the higher group by discarding half of the farms with inputs below the average. Hence, although there are 31 farms shown on each of the separate groups, there are only 43 shown in Figure 16.

roughly the relation of the corn fed to the pork produced. Comparing this chart with the two others, the reason for the difference in the slope of the regression lines is apparent; the regression line for the heavier feeding tended to be parallel to the upper part of the curve, and the one for lighter feeding to the lower part of the curve. The slope of a regression line is therefore a poor measure of the effect of input upon output, as it will change, depending upon the part of the curve where the bulk of the records happen to fall in any given sample.

The relation shown by the curve in Figure 16 is of much more value than that shown by the regression lines in either of the other charts. The curve gives an approach to that exact measurement of diminishing returns which has been shown to be essential to any detailed input-output analysis.

A method for determining the exact curve from the group averages has been evolved by Dr. W. J. Spillman. This method gives a rather satisfactory way of determining the curve in cases of simple two-variable relationship such as this. A measure of the accuracy of the results, comparable with the coefficient of correlation, can likewise be obtained for such curvilinear relations.[22]

MULTIPLE CURVILINEAR CORRELATION

The curve in Figure 16 showing the relation between corn fed and pork produced is still subject, however, to one limitation that was stated with regard to the straight line in the case of simple correlation. It tells, it is true, how much gain in weight per pig was found for the average of the droves receiving each stated input of corn; but it does not tell how much of this gain was due to the corn alone, and how much was due to the other feeds the pigs received. "Other things" are still not constant.

An extension of the method of multiple correlation for linear relations is applicable to the solution of this problem. The use of the multiple curvilinear analysis enables one to determine approximately the curve of diminishing returns for each variable separately, and supplies a means of making estimates of what the gains will be for any given combination of inputs, taking into account the location of each input on its curve of diminishing returns. The reliability of the curves for each variable and the accuracy with which the output may be estimated, can also be determined by this method.[23]

Even the method of multiple curvilinear correlation will not solve all the difficulties met in input-output analysis. Many problems involve not merely determining the net effect of individual inputs, but determining the effect of one input in the presence or absence of others, or with specified variations in others. How to handle these more complex relations has yet to be worked out. As the simpler phases of input-output analysis are attacked and conquered, the more difficult problems will be disposed of with increasing facility.

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[22] References on simple curvilinear correlation:
Spillman, W. J. Application of the Law of Diminishing Returns to Some Fertilizer and Feed Data. Journal of Farm Economics, Vol. V, No. 1, pp. 36–52, January, 1923.
Spillman, W. J., and Lang, E. The Law of Diminishing Returns. 1924, pp. 12–17, 70–75.
Ezekiel, Mordecai. A Method of Handling Curvilinear Correlation for any Number of Variables. (Part I), Journal of the American Statistical Association, Vol. XIX, New Series, No. 148, 1924.

[23] Reference on curvilinear multiple correlation: Ezekiel, Mordecai. A Method of Handling Curvilinear Correlation for any Number of Variables. (Part II), Journal of the American Statistical Association, Vol. XIX, New Series, No. 148, 1924.